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How Student Positioning Can Lead to Failure in Inquiry-based Classrooms

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ABSTRACT

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In an effort to improve mathematics instruction, I studied how the positions students choose or are pressed to take on can lead to failure in an inquiry-based classroom. Positioning refers to the temporary roles that a student or a teacher can place each other in or take on in a given classroom environment. Since positions are constructed and maintained by language use, I drew from positioning theory and discourse theory for my theoretical framework. From analyzing transcripts of classroom episodes and interviews with the teacher and a student, I identified common positions that the teacher and the student took on. I also looked at how these positions were negotiated in the classroom. I give a specific example of how positions could negatively change between the teacher and the student. The teacher went from a position that supported inquiry-based instruction to a position that supported traditional instruction, and the student went from a position that supported learning to a position that was detrimental to learning. The positions and the negotiation of positions impacted learning and helped to explain why the student might be failing. I found that positions changed (for better or worse) because of specific pressures, which I analyzed. For instance, when the teacher and the student succumbed to specific pressures and stepped out of certain positions, the teacher felt ineffective and the student engaged in less desirable positions that hinder learning. From this study, we can learn how to better address student and teacher failure in an inquiry-based classroom. These results have significant implications for research, teaching, and teacher education. Therefore, positioning is vital to the implementation of inquiry-based instruction in that it can either maintain or undermine it.

Keywords: [positioning, discourse, failure, inquiry-based instruction]

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In our current society, there are many national policies that promote the idea of student proficiency in mathematics (e.g. "No Child Left Behind Act of 2001," 2001). Many of these policies emerged because studies like TIMSS (Gonzales, et al., 2009) have shown that the United States is behind in student proficiency compared to other countries. If students are not proficient according to letter grades, test scores, and/or teachers' opinions, then they are commonly labeled as failures. The national standard, as set forth by the National Council of Teachers of Mathematics (2000) and laws like No Child Left Behind ("No Child Left Behind Act of 2001," 2001), is that all students should be proficient in mathematics, not just a majority. As a greatly debated issue, mathematics education has yet to reach high levels of proficiency.

One area of mathematics education that has been hotly debated is instruction. Currently, there seem to be two broad schools of thought concerning how mathematics should be taught. Advocates of traditional mathematics instruction believe that mathematics is a set of rules and procedures, that the best way to teach mathematics is to provide and model the knowledge that students need to know, and that students learn best by memorizing the information and replicating what the teacher does (National Council of Teachers of Mathematics., 2000; Smith, 1996). On the other hand, advocates of inquiry-based instruction believe that mathematics "is a dynamic body of knowledge that is continually enriched through conjectures, exploration, analysis, and proof" (Smith, 1996, p. 393), that the best way to teach mathematics is to create conditions and pose problems that engage students in doing mathematical activities, and that students learn best by exploring, conjecturing, and questioning assumptions (Smith, 1996). Even though there are students who fail with both types of instruction, there is better documentation of

student failure in a traditional mathematics classroom (Boaler, 2002; Schoenfeld, 1988; Skemp, 1978; Sowder, 1988; Thompson, Philipp, Thompson, & Boyd, 1994).

In past studies, authors have focused on cognition as a lens to view student failure (Boaler, 2002; Schoenfeld, 1988; Skemp, 1978; Sowder, 1988; Thompson, et al., 1994). However, the issue of student failure is not only influenced by what a student knows but also influenced by the student's social identities (Gee, 2005), which are the "different ways of participating in different sorts of social groups, cultures, and institutions" (p. 1) both in and outside the classroom. Social identities are built through social interactions within these contexts. Because is it less understood how social interactions can lead to student failure, I am viewing student failure through the lens of positioning. Based on the work of Harre and Langenhove (1999), positioning refers to how a student is cast by the teacher or casts him or herself in a given classroom environment. In any particular position, students are expected to act and to be acted upon in particular ways. For example, a student who is positioned or positions themselves as a *misbehaver* might be expected to work against the teacher and to be disciplined by the teacher. By understanding this interaction and how the student came to be so positioned, I can make better sense of student failure in a mathematics classroom. Therefore, positioning adds more insight into student failure than theories like cognition alone.

Failure in Traditional Mathematics

Throughout the literature on student failure in traditional mathematics, there is evidence that the way students are positioned in traditional mathematics leads to student failure. As a note, these positions are my own constructs and were not identified by the following authors, but the descriptions were rich enough for me to infer these positions. Authors like Lampert (1990), Hannula (2002), Schoenfeld (1988), Smith (1996), and Gregg (1995) have all discussed student

failure in traditional mathematics. In the next few paragraphs, I identify some of the positions that can be inferred from papers on traditional instruction and connect this to student failure.

When a teacher tells students that they are wrong, students in certain positions can respond in negative ways. Lampert (1990) described the consequence of what I inferred as students being positioned as *computational learners of mathematics* in a traditional classroom. Students in this position are expected to solve problems by applying known formulas and providing answers. For this position, students sometimes connect their self image to their answers because they have invested time and effort into their work, making the answer a product of their work. So when they are told they are wrong, they may feel this implies there is something wrong with them personally. Other students may disconnect themselves from their answer in order to preserve a positive self image. As a result, students may position themselves as *non-doers of mathematics*. For example, a student named Rita in Hannula (2002) positioned herself as a non-doer of mathematics in order to preserve her positive self image. This way when Rita struggled with the mathematics she could claim that she was not stupid, the math was. Even though this protected her positive self image, it also meant that she would quit trying. Such a response is not uncommon with struggling students, as documented in Horn (2008). If students feel devalued or quit trying, this could lead to student failure.

Also, in Schoenfeld's (1988) work, I can infer another position that students take up in traditional mathematics. The student position of *key word detector* in a traditional classroom implies that the student looks for the key words in the problem to know what to do to produce an answer as opposed to reasoning about what the question is really asking. This position led 29% of students in the third NAEP (as cited in Schoenfeld, 1988), National Assessment of Educational Progress, to conclude that the number of buses needed to transport soldiers was 31

remainder 12. When students are so focused on procedures, it is hard for them to gain a conceptual understanding of the content (Skemp, 1978). Without the conceptual understanding, students may not be able to extend the mathematics they learn to other mathematics classes and to mathematics problem outside the classroom. Therefore, this position is harmful to students.

Another example of a student position in traditional instruction that may contribute to student failure is *knowledge receiver* in relation to a teacher position of *knowledge giver* (Smith, 1996). In this case, the teacher's position of *knowledge giver* implies that he or she holds all the answers, is always right, and corrects students when they are wrong. The student's position as *knowledge receiver* implies that he or she will take note of what the teacher is saying and memorize the given information. From these positions, students learn that mathematics is a set list of rules and procedures. They are trained that learning mathematics means listening to the teacher and modeling his or her actions. They learn that the teacher and the textbook are the mathematical authorities in the classroom and that those are the sources to turn to for validation. With the understanding of what mathematics is, what mathematical learning is, and what mathematical teaching is, students learn well that mathematical knowledge is unattainable unless someone is there to provide them with it. If students are unable to make sense of what their authority source is saying, students may come to view mathematics as unattainable, which could lead to failure. Another possible result of the *knowledge receiver* position could be that students stop thinking, stop reasoning, and stop questioning assumptions because of their dependence on the teacher. When students do not view mathematics as a sense making activity, mathematics may lose value to those students or they may be unable to do tasks that require them to think for themselves, which may lead to failure.

Another set of positions that students take up in a traditional classroom can be inferred from Gregg's (1995) work. Within traditional instruction (both in upper and lower track mathematics courses), a common feature is the initiate, respond, and evaluate (IRE) pattern used in class discussions (Gregg, 1995). A teacher initiates, usually by asking a question, and students are expected to reply. Then the teacher evaluates that response immediately. In this case, the teacher takes the position of *judge* while the students are relegated to the position of *defendant*. From this position, students learn that only the teacher knows the truth of mathematics. So students may learn that to do mathematics, one must be dependent on an outside source, like the teacher or a textbook. Therefore, if the students cannot comprehend the teacher or textbook, students may feel that the mathematics is unattainable and may quit trying to learn, leading to failure.

Gregg (1995) studied traditional instruction in an upper track class and a lower track class, and his work was descriptive enough for me to infer additional positions that were related to failure. In the upper track class, students were often positioned as *menial contributors* to a class discussion because they would bring up mathematical issues that the teacher would label as unimportant technicalities. This can teach students that they cannot know what is mathematically important on their own or that they are expected to already know the mathematics they are questioning. Hence, students learn that they are not as smart as the teacher. In the lower track class, students were frequently positioned as *unmotivated learners*. The teacher used a system of rewards to motivate her upper track students but not with her lower track students because she felt that those students did not care. Many times, this teacher tried to motivate her students by using her authority over them. She would demand that students perform a task strictly because she was the teacher and they were the students. In this situation,

students were cast in an antagonistic position to the teacher, which allowed the teacher to avoid taking responsibility for whether or not they learned. Further, the teacher was dehumanizing students since their interests, feelings, and characters were not being honored. Both results can lead to student failure.

Overall, there are many examples of positions (that can be inferred) that lead to student failure in the literature. These positions affect how students view themselves, the mathematics, the learning, and the teaching. As a whole, all of these positions have something in common. They seem to position students as *outsiders to mathematics*, which ultimately contributes to student failure.

Moving Past Traditional Instruction

Because traditional instruction is problematic to many students, some curriculum developers believed that it was the teacher who directly contributed to student failure. Their solution was a teacher-proof curriculum where the teacher was no longer providing mathematical instruction. Instead students were given a series of pre-tests, assignments, and post-tests to teach them mathematics. However, the case of Benny (Erlwanger, 1973) demonstrated that a teacher-proof curriculum was ineffective. Benny was positioned as a *successful student* and yet he constructed many misconceptions which were validated by his ability to pass the exams. Therefore, a teacher-proof curriculum can position students to fail as well.

In response to the student failure in traditional instruction, inquiry-based instruction was developed, which still poses problems for mathematics teaching and learning. Inquiry-based instruction is meant to be more student-centered and does this by building off of and working with the student's understanding of mathematics. It also includes giving students an opportunity to do mathematics and not just to memorize it. Further, inquiry-based mathematics focuses on

collaboration and discussion as a way to understand mathematics. There has been a great deal of debate over whether inquiry-based instruction is better than traditional instruction. Much of the current literature attempts to promote inquiry and/ or show that it does not do any more harm than traditional mathematics instruction (Boaler, 2002; Hiebert, 2003). However, even with the benefits of inquiry and the possibility that it does no more harm to students than traditional, there are still students who fail. From the literature on inquiry-based instruction, there is not much information on how student positioning leads to student failure since most authors talk about how students were positioned positively in inquiry. Are these positions really positive or are they just perceived as positive? Some of these "positive positions" could actually be contributing to student failure, which I discuss in more depth in the next chapter. Therefore, we need to be more critical of these positive student positions. For now, we can examine the reasons the authors express for why students fail in an inquiry-based classroom.

Among various authors, there is great diversity in the reasons why students fail in an inquiry-based classroom. Hiebert (2003) claimed that student failure is a direct result of the inquiry-based instruction not being implemented as it was intended. However, even with instruction close to the ideal of inquiry, there are still students who fail (Ball, 1993; Boaler, 2002; Lampert, 1990). Other authors and teachers may choose to blame student behavior or the difficulty of the content. Boaler (2002) suggested that the students may fail because of the time they spend off task. Despite these failures and their causes, mathematics educators that advocate inquiry-based instruction tend to believe that all students are capable of learning mathematics given the right circumstances and setting (Ball, 1993; National Council of Teachers of Mathematics, 2000), making *non-doers of mathematics* a traditional student position that has no place in inquiry.

In the majority of the literature that tries to explain student failure in inquiry, authors account for student failure by turning to the student's previous education and roles in traditional mathematics (Ball, 1993; Boaler, 2002; Hannula, 2002; Lampert, 1990; Smith, 1996). From the way these authors talked about student failure, I inferred that these authors saw students as positively positioned by the teacher in inquiry, but because of the negative positions they learned in traditional settings, they struggled in inquiry settings. In Hannula (2002), Rita's experience with traditional positions, especially *non-doer of mathematics*, led her to expect an unpleasant experience in any mathematics classroom. Rita's resistance to mathematics contributed to her initial failure in the inquiry-based classroom.

Boaler (2002) mentioned that some students resisted inquiry because they did not like the freedom they had compared to the traditional setting. Because inquiry-based positions are so different from traditional positions, it is unfamiliar territory for many students (Smith, 1996). The unfamiliar and unknown may contribute to student discomfort, and so they resist the new type of instruction and the new positions. When the teacher of inquiry does not fulfill his or her position as the *teller* (a teacher position in traditional mathematics) by telling a student if they are right or wrong or by giving direct instruction, some students may become frustrated with the teacher or uncertain about their solutions (Ball, 1993; Lampert, 1990). They are used to the teacher taking on the position of *teller*. Since the teacher is not doing so, the student may feel discomfort and resist the inquiry-based instruction. One reason Lampert (1990) gave for why students fail in inquiry-based classrooms was that students in traditional mathematics are used to engaging in silent activities. Since discussion is a key element of inquiry, silence may be detrimental to student learning. Therefore, a student's previous experience in traditional

instruction may make him or her unwilling or unable to take on or accept the new positions in inquiry which influences his or her performance in inquiry-based instruction.

Overall, positioning plays a primary role in the issue of student failure. In traditional classrooms, students can take on positions that lead them to fail. Because of the student positions in traditional classrooms, students in an inquiry-based classroom struggle to take on the new positions associated with this new type of instruction. Many authors have tried to use a student's traditional background as a means to explain the less than 100% pass rate in an inquiry-based classroom but this topic has not been the focus of any of their studies. Further, these authors focus almost exclusively on the positive positions occurring in the inquiry-based classrooms. So the problem is that we know student's fail in inquiry-based classrooms, but we do not know the extent to which student positioning affects this failure or what positions they might be taking on as they fail. To address this issue, my study examines the common positions that failing students take on in an inquiry-based classroom. In chapter 2, I discuss positioning theory in more depth and build it up from Gee's (2005) theory of discourse. In chapter 3, I outline and justify the methods I used to collect and analyze data. In chapter 4, I discuss the common student and teacher positions that I found in an inquiry-based classroom and examine the negotiation of student and teacher positions. In chapter 5, I discuss the factors that press the student and the teacher to change positions. In chapter 6, I discuss the implications of my study and the areas for future research.

Chapter 2: Discourse and Positioning Theory and An Examination of Student Failure in Inquiry-Based Mathematics

Discourse and Positioning Theory

When it comes to positioning theory, Harre and Langenhove (1999) are the authorities on the subject. They talk about how people position themselves and others through language. However, to make sense of and to understand the meaning of a particular piece of language, one not only has to consider what was said but also the source from which it came and the accompanying gestures. Inside jokes and sarcasm are good examples. When someone says something, a person can tell if he is joking from his or her knowledge of that person (does he use sarcasm) and also the smile on his face. Because Harre and Langenhove's concept of positioning is only based on verbal language, it does not fully account for the complexities of human interactions. However, Gee's (2005) theory on discourse does. When I build Harre and Langenhove's (1999) positioning theory up from the foundation of Gee's (2005) discourse theory, I can more accurately identify positions occurring in social interactions because I have other context clues. In the next section, I will first discuss Gee's (2005) theory of discourse, then discuss Harre and Langenhove's (1999) theory of positioning, and finally discuss the connections between the two. After I establish the theories behind my study, I will discuss some examples of positioning in discourse.

Gee's discourse theory. By nature, mathematics classrooms are social settings or contexts. Teachers and students talk and interact with each other. Each student, and the teacher, are involved and have been involved in other social interactions where they are or were expected to or expect other to speak, act, dress, and hold similar values as other members of the social setting. They have come to understand what it means to learn mathematics in school, what it

means to be a mathematics student, and what it means to teach mathematics. Both teachers and students are expected to and expect each other to speak and act in particular ways. Therefore, there is more than just language that characterizes and gives meaning to human interactions in a social setting.

Gee's (2005) theory of discourse attempts to incorporate all of these additional elements that give meaning to human action. He does this by making the distinction that discourse, with a lower case d, refers to only the language in use and that Discourse, with a capital D, refers to the language in use as well as other important elements such as apparel, gestures, tools (symbols and representations), values, and beliefs. Each of the social interactions an individual is involved in defines and is defined by a set of Discourses. It is possible to recognize that a person is engaging in a particular Discourse by the way they talk, act, dress, and believe.

All of the components of Discourse (the language in use, the apparel, the gestures, the tools, the values, and the beliefs) create an identity kit which serves to define "who's-doing-what." The "who" in this case represents a socially situated identity. We enact different identities through different Discourses. A person talking to his or her parents may adopt a more formal identity than when he or she is talking to a friend (Gee, 2005). The "what" represents a socially situated activity. Each member of the Discourse is expected and expects others to engage in certain activities while enacting certain identities. For example, in a traditional mathematics classroom, the students may expect the teacher to lecture the entire class period because that is what mathematics teachers do.

In any social interaction, Discourses serve the dual purposes (Gee, 2005) of (1) accomplishing social tasks and (2) conveying a particular perspective. So again, the purpose is to identify and be identified as "who-doing-what." Most of the time, this is a subconscious act.

When we engage in conversation, we do not always consciously think about the purposes of the language we are using. Even if we do not consciously think about it, everything that we say, with its accompanying actions, serves a specific purpose.

Based on our experience with certain Discourses, we begin to recognize patterns and develop Discourse models. Discourse models are "the largely unconscious theories we hold that help us makes sense … of the world" (Gee, 2005, p. 70). So, everything we say, do, and believe makes sense to us through our models of the world, and we try to make sense of what other people say, do, and believe through these models. For example, students develop a Discourse model of what it means to do mathematics in the Discourse of traditional school mathematics. In this model, students come to view mathematics as a set of rules and procedures. Some students even view mathematics as something that must be demonstrated to them before they can do it. So when a non-traditional teacher asks the students to solve an unfamiliar problem without demonstrating how to do it first, their Discourse models tells them that they cannot solve the problem. So, students have Discourse models that help them make sense of the world in which they live. Discourse models may be partial or inconsistent as they can incorporate conflicting social and Discourse values. This reflects our diverse and unique experiences from which we build our models to better understand the world we experience. In summary, students build Discourse models based on their experience with various Discourses in and out of school.

Positioning theory. Now that we have the foundation of discourse theory, we will look at positioning theory and then draw connections between positioning and Discourse. According to Harre and Langenhove's (1999) conception of positioning, positioning is a discursive act where participants in the discourse take on or are assigned certain roles or parts so that their actions and speech can be interpreted. Most often, people are completely unaware that they are

positioning themselves and others and being positioned by others. The interpretation, recognition, and enactment of certain perceived positions stems from the history of the participants and of the perceived positions available in a particular social interaction. Positions are socially constructed because people in particular social interactions are assigned or choose positions to carry out some social activity. There may be similar positions in different social interactions, but positions are unique to the social interaction in which they occur. For example, you can recognize a person's position in a company by interpreting his or her actions by comparing those actions to your socio-historical knowledge of how people generally act in various positions of a company. This assumes that you have access to this historical knowledge; otherwise, this position would be unfamiliar to you. In summary, one can recognize the position in a company because of his or her access to social interactions involving that position in some manner.

Positions are also determined and influenced by positions of others in that social interaction. The relationships between these positions are called social relationships. For example, if one person positions him or herself in a position of power, then he or she positions another as a subordinate. So each position in a social interaction is related to and is influenced by each other.

Positions are embedded in storylines, which are socially accepted discursive forms. Storylines, like positions, have been socially constructed by the participants of a specific social interaction. Storylines are like informal scripts of a play. They inform each character of when and how to participate in the particular ongoing social activity. Each character takes on a certain role, but these roles change when the script changes. Storylines change as the purpose for the language in use changes. The participants in the storyline must also take up new positions in

order to carry out the new purposes of the storyline. Within each storyline, there are various perceived positions available to carry out that purpose. When people position themselves or others, they are signaling to the participants that they are following a certain storyline. Therefore, positions and storylines are interrelated. However, how do the participants indicate or recognize the storylines or positions? Harre and Langenhove (1999) suggest that positions and storylines are connected to language in use. Speech acts serve to signal to the participants specific storylines and positions. The following triad (Figure 1) represents this concept.

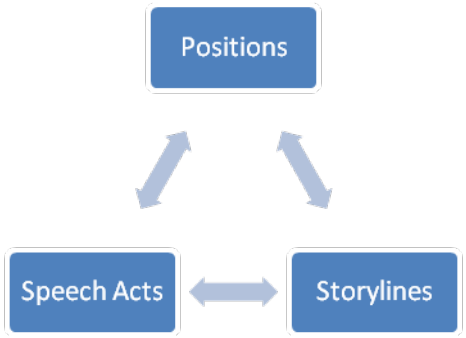

Figure 1. Positioning triad shows how positions, storylines, and speech acts are interrelated.

Connections. Since storylines are discursive forms, they are situated within Discourses which suggest that there are other elements of Discourse, besides speech acts, that participants might also draw upon to construct and interpret positions and storylines like apparel, gestures, facial expressions, physical orientation, tools (symbols and representations), and values. Because storylines are constituted by the elements of Discourse, they are always situated within a Discourse. This also suggests that Gee's (2005) purposes for Discourses also apply to storylines. So storylines, as discursive forms, serve to (1) accomplish social tasks and (2) convey a particular perspective. Thus we have "who's-doing-what" within storylines. The "who", which Gee defined as a socially situated identity, here refers to the positions available for participants to take on. The "what" is the specific activity in the storyline. Therefore, positions are socially situated identities doing certain activities within storylines. In order to more

accurately identify and make sense of the storylines and positions, we need access to the participant's Discourse models which are his or her theories of how the world works based on their previous experience with other storylines and positions. Therefore, discourse theory provides a foundation on which to build positioning theory.

Examples. To illustrate how positions and storylines may be constructed by and influence Discourse, consider the following example of two students talking in a mathematics class. Even though there is more to Discourse than speech acts in a storyline, my example just consists of speech acts for the sake of simplicity.

Student 1: How did you do number 3?

Student 2: I subtracted 1 from both sides and divided by 2.

Student 1: So, you got x=5?

Student 2: No. When you subtract 1, what do you get?

Student 1: 8.

Student 2: When you divide by 2, what do you get?

Student 1: 4?

Student 2: Good.

Student 1: How did you do number 5?

These two students are talking in the Discourse of traditional school mathematics. This is an example of a storyline that is available in this Discourse. It is not surprising that these students are engaging in a traditional school mathematics storyline because this is a type of storyline that students have learned to engage in for a mathematics classroom, so this storyline and the related positions have a socio-historical setting within a mathematics classroom. The storyline in this situation seems to be that of a teacher helping a pupil get an answer, where student 2 is positioned as the teacher and student 1 is positioned as the pupil. It is evident that

student 2 is taking on the position of teacher because he is the one answering the questions with "yes," "no," and evaluations like "good" and posing leading questions for the other student. It is evident that student 1 is taking on the position of pupil because he is the one asking specific questions to get an answer as well as answering the leading questions of the teacher. The purpose of this exchange is for student 1 to produce the right answer to the problem. Therefore the speech acts, storyline, and positions are all interrelated, serve a specific purpose, and are situated in a socio-historical context. The storyline could have been different if student 1 had been debating with student 2 over his mathematical thinking. In that case, the students may have been positioned or positioned each other as collaborators in a storyline of checking answers.

In order for positioning to occur, the participants in the social interaction have to acknowledge the position. Positions are not always acknowledged because they may be new and unfamiliar. The participant may not have access to the historical context of the position to refer to, so the participant may retain a familiar position or try to take on the new position by modifying a familiar position. Participants may even be resistant to the new positions, which may be particularly true in cases where the participant is new to a Discourse. The following example illustrates this point. In this example, the student and the teacher are trying to position each other and themselves in particular ways that may not be recognized or accepted by the other. The student here is enacting the traditional school mathematics Discourse, while the teacher is trying to enact an inquiry-based mathematics Discourse. We can make sense of this conflict by considering the student's and teacher's Discourse model of school mathematics.

Student: Is 5 the correct answer?

Teacher: What do you think?

Student: This is what I did. Is that correct?

Teacher: You tell me.

In this case, the student and the teacher could be perceiving different storylines and positions since they are enacting different Discourses. The student, by enacting the traditional school mathematics Discourse, could see the storyline as the teacher helping a student get the right answer where the teacher has the position of *evaluator* (the teacher is the final say on what is right and what is wrong), and the student has the position of *answer checker* (the student turns to the teacher to find out what answer the teacher expects). This is evident from the speech acts, which fall into the traditional school mathematics Discourse. In the student's Discourse model of school mathematics, the correctness of an answer needs to be confirmed by an external authority, and it is the job of the teacher to act as the external authority.

On the other hand, the teacher, trying to enact an inquiry-based mathematics Discourse, could see the storyline as the student verifying his or her answer where the teacher has the position of *observer of student problem solving* and the student has the position of *mathematical problem solver*. The position of *observer* implies that the teacher is trying to assess student understanding as well as give students an opportunity to solve problems without teacher intervention. The position of *problem solver* implies that the student is trying to solve problems independent of the teacher. Through the speech acts (which are embedded in inquiry-based mathematics Discourse), the teacher is trying to enact this storyline and positions. In the teacher's Discourse model of school mathematics, teachers must avoid passing judgments on student solutions so that students learn to reason about the correctness of their solutions. Because of their differing Discourse models, neither the student nor the teacher is willing or able to accept the storyline and position of the other.

Work on Discourse (Gee, 2005) and positioning (Harre & Langenhove, 1999) have focused on stable Discourses, positioning, and storylines. In my study, I pushed beyond that to

see how Discourses, positions and storylines are not stable. People are always in a Discourse or storyline or position. However, there are times when Discourses, storylines, and positions are in flux as the participants are trying to figure out which storyline to follow or which position to take on. The participants may realize that they are not in a position they want to be in or should be in, and so they might try to reposition themselves. In the last example, we looked at the conflict that may arise between the teacher and student. However, there is another way that teachers and students might react. At the beginning of any school year, the student is trying to figure out what the teacher is doing and how to play along. A person plays along by learning how it is appropriate to talk, to behave, and/or to view mathematics. For some cases, both the teachers and the students are trying to figure each other out at the same time. Thus, at the beginning of the school year, students and teachers are often trying to figure out the storylines and positions to engage in with each other.

The following is a progression that leads to the positioning of the students and to students learning to position themselves. First, either the student or the teacher chooses a storyline and position for his or herself and the other participants of the discourse. For the sake of clarity, consider the case of the teacher making the first discourse move by choosing the storyline and positions. Second, the students interpret this storyline, which may take quite a few speech acts. They decide, through their Discourse models, on what the storyline is and what position the teacher is placing him or herself in as well as the students. They may interpret it in the same way as the teacher or in a different way, as in the example above. A different interpretation may stem from the students' previous experience with positions in traditional mathematics classrooms, which is part of their Discourse models. The position the teacher is trying to urge the students to take on may be inaccessible to them, due to previous experience or may be undesirable to the

student. Third, the students choose positions to take on, which may change as the storyline develops. These positions can be in compliance with the teacher (accepting the perceived position the teacher is offering) or in opposition to the teacher (rejecting the position the teacher puts upon them, and instead trying to change the storyline like the student in the previous example). Some students may take on less desirable positions that may devalue them in order to comply with authority, which is part of their Discourse models. Other students may struggle to find a position they can take on that fits with the ongoing speech acts, gestures, facial expression, and other communicative elements of the interaction. After the student takes on a position (which may support or challenge the teacher's position) and chooses a storyline to follow, the student positions the teacher and so the teacher has to decide how to position himself in response just as the student had to. This cycle of teacher and student positioning continues throughout the duration of a class period. Overall, this progression and cycle may not be a conscious act of the students or teacher and will occur continually as each new storyline, and its related positions, is introduced to the participants of the Discourse.

With this model based on Discourse and positioning, I can better make sense of students' and teacher's words and actions, which will shed light on the problem of student failure in inquiry-based classrooms. The positions that I hypothesize about from the storyline, speech acts, and other elements of Discourse may indicate that a failing student is positioned as a failure, that the failing student is unwilling or unable to take on the position put forth by the teacher, or that the student is taking on a less desirable position. In any case, knowing common positions that lead to student failure in an inquiry-based classroom may help us to refine our teaching practice.

Examination of Student Failure in Inquiry-Based Mathematics

In the current literature on inquiry-based mathematics, authors tend to focus on the positive student positions and the related storylines, which seem to dominate these classrooms, in relation to the negative student positions of traditional instruction (Ball, 1993; Boaler, 2002; Gregg, 1995; Lampert, 1990; Smith, 1996). Even though they do not talk about students through the lens of positioning theory, there is enough information in each article for me to hypothesize about the possible positive positions and the related storylines as well as the possible negative positions and the related storylines that could be occurring. For example, in Smith (1996), I inferred the traditional teacher position of *knowledge giver* and student position of *knowledge receiver* in the storyline of teacher telling students what they need to know about mathematics in juxtaposition to the inquiry-based teacher position of *facilitator* and student position of *doer of mathematics* in the storyline of students drawing mathematics conclusions from problems posed by the teacher. This contrast serves to show how inquiry-based positions and storylines are better than traditional positions.

Another reason these authors may focus on the positive positions and storylines of inquiry-based instruction may be because positive positions and storylines are predominant in these social settings or because of their focus is only on the harmful positioning and storylines in traditional mathematics. When authors focus only on the positive positions and the related storylines of inquiry, they may miss some of the reasons that students are struggling in inquiry-based classrooms both related and unrelated to traditional positions.

In Lampert (1990), the teacher is described in such a way that might be consistent with the label of the positive position of *facilitator* or *guide*. During a class discussion, this means that the teacher contributes to help clarify students' ideas, slow down the pace, or prompt students to consider a different case in the storyline of students drawing mathematical

conclusions from problems posed by the teacher. However, the students may interpret the teacher's actions as one in the position of *manipulator* as the teacher tries to take the discussion in a direction he or she thinks is worthwhile. The storyline in this case is the teacher working to get the students to come to a particular mathematical conclusion. Another way that the *facilitator* position could be interpreted as a *manipulator* position is when the teacher asks students to present their work at the board. Usually, these students are chosen with a specific intent, which again manipulates the direction of the mathematics. When the teacher is perceived as a *manipulator*, students may perceive mathematics to be a guessing game of what the teacher is looking for. As students try, or do not try, to find what the teacher is looking for, they can stop thinking for themselves, which may lead to failure.

Both Boaler (2002) and Ball (1993) present the positive student position of *mathematical expert* in the storyline of students determining the validity of mathematical concepts. In this position, the students become the verifier of their own mathematical work as they defend and discuss their solutions. However, since the teacher is the one who chooses the problem and even the direction of the discussion, the student does not have as much power as it may seem. Instead of *guide*, the teacher's position could be *gate keeper* to the mathematics where students can only pass through if the teacher allows them. This could be very frustrating to students who wish to explore the mathematics more or move on. Some students may stop using initiative, wait for the teacher, and exert minimal effort, which can lead to failure. In this situation, the storyline is students waiting for the teacher to introduce new mathematical concepts. Another problem with the positive student position of *mathematical expert* is that students may interpret this as the traditional teacher position of *mathematical expert* where the teacher not only determines validity of mathematical concepts but also evaluates students' answers in the storyline of the

teacher being the unquestionable source of all mathematical knowledge. In this way, some students not only engage in validating their own mathematics but also in evaluating the answers of their peers. This can be intimidating to peers. Criticism from a peer can be more harmful because peers are less concerned with the emotions of another student. Some students may try to cope with this by disengaging from the mathematics and other students may not be able to cope. In either case, student failure may result. Therefore, *mathematical expert* can be a negative student position.

Throughout the literature concerning inquiry-based mathematics, students are commonly positioned as *doers of mathematics* or *mathematical problem solvers* in the storyline of students solving problems with minimal teacher interference (Ball, 1993; Boaler, 2002; Gregg, 1995; Lampert, 1990; Smith, 1996). In this case, students are left to reason and question assumptions with very little if any teacher intervention. However, students may interpret their position to be *grand inquisitor* which implies that they need to challenge and question everything in the storyline of students challenging facts and assumptions. For example, Ball (1993) described a student who decided to question the definition of prime numbers. Definitions in mathematics demonstrate a widely accepted convention and so questioning a convention may be unproductive for students as it draws their attention away from more important mathematical concepts, which may lead to failure. The confrontational approach of the *grand inquisitor* may intimidate other students, making them less willing to offer their ideas, making them *spectators* of the sport of mathematics, and thus harm their mathematical growth and possibly causing them to fail. The storyline of the *spectator* position is some students observing or waiting for other students to come to the mathematical conclusions. Observing mathematics is not the same as engaging in

mathematics, which is a goal of inquiry-based classrooms. Therefore, this may seem like a positive position when in fact it can be harmful for that student and those around him or her.

Overall, in an inquiry-based classroom, we only have a limited idea of what positions are available for students to take on or are offered to students and how these perceived positions are related to student failure. The positions that I identified in the literature were hypothetical but allowed me to analyze potential sources for student failure. If I can identify common positions that cause students to fail through this study, then maybe we as mathematics educators can better address student failure in an inquiry-based classroom by renegotiating positions available to students. Because of the lack of criticism of positive student positioning and the gap in research on negative student positioning, there are still questions that need to be addressed. First, for students failing in an inquiry-based classroom, how are the students positioned or how are they positioning themselves? Second, how are positions taken on related to student failure in the inquiry-based classroom?

Chapter 3: Methodology

Now that I have elaborated on how the problem is situated and what my theoretical constructs are, I describe and explain how I worked towards answering my research questions. My choice of methodology was determined by the qualitative, ethnographic, and discursive nature of the research questions. The purpose of this methodology section is to be very explicit about the setting for my study, my data collection methods, and my data analysis. If I have an appropriate setting, then I can collect pertinent data. If I have pertinent data, then I can analyze my data and find answers to my research questions. In summary, this chapter serves to describe and justify my choices in the design and execution of my study.

Setting

To answer my research questions, I looked in an environment where I was most likely to see the phenomenon I was interested in studying (Honigmann, 1986; Pirie, 1998). The phenomenon I was interested in studying was how a struggling or failing student positioned him or herself or was positioned in an inquiry-based classroom. Further, I carefully chose my subjects in order to observe the phenomena I was interested in examining. By choosing a setting with particular characteristics, I captured the phenomena occurring in an inquiry-based classroom and worked to answer my questions about the common types of positions.

First, I identified and accessed a classroom environment that was a good representation of inquiry-based instruction. I assumed that a single inquiry-based classroom shared some common storylines and positions found in most inquiry-based classrooms (Merriam, 2002; VanWynsberghe & Khan, 2007). Sampling from across inquiry-based classrooms would have been ideal but was beyond the scope of a master's thesis. The results of this study may lead to broader studies in the future, but for this thesis, I focused on one classroom. To choose an

inquiry-based classroom, I enlisted the aid of a university professor in my department who spends a significant amount of time out in the public school with mathematics teachers and who is familiar with the characteristics of inquiry-based instruction. In order to find an inquiry-based classroom, I asked this university professor for a classroom with the following characteristics:

- The teacher does very little lecturing
- The teacher has students discuss mathematics as a class and in smaller groups
- The teacher does not provide students with formulas without the students thinking about the problem first
- The teacher encourages students to conjecture and contribute their ideas
- The teacher is more interested in how students think than what answers they produce
- The tasks student engage in are more problem solving and less strict computation

Since it is through speech acts and other elements of the Discourse that positions are established, I needed to collect data that was embedded in the Discourse of inquiry-based mathematics. After receiving a list of possible candidates, I found a teacher of an Algebra 1 class who was willing to participate in the hopes of improving his own teaching. I gave the teacher the pseudonym of Mr. Green.

Second, with the help of the teacher, I went into his classroom before I started gathering data to choose one of his Algebra 1classes that would be suitable for my study. In the first class I attended, there was not much student talking and interacting. I needed a class with lots of dialogue between students so that I could collect enough speech acts and other relevant information to hypothesize about positions. When I attended another class, there was much more student talking and interacting. Even the teacher described this class as his most talkative class.

I then asked the students of that class to participate by telling them that I was interested in their experience as mathematic students. By so doing, I found a suitable environment for my study.

Third, I worked towards finding my two student participants who were "failing" in the inquiry-based classroom. During the initial observations of my study, I tried to look for students who exhibited failing behaviors such as openly resisting the teacher by questioning the teacher's actions, not responding in desirable ways, turning in assignments late consistently, or getting low grades. I started with a list of students with the lowest grades in the class and then looked for students who exhibited some of these other behaviors. By consulting with the teacher and taking note of students during my own observations, I chose three students to focus on.

When I informally asked them to participate in interviews and journal writing, two students agreed to participate and were willing to tell me their honest opinions. The first student I chose for her seeming lack of participation in the mathematical activities, failure to turn in assignments, and social involvement with her peers, to which her teacher attributed her low grade. Despite her lack of participation, she was still vocal enough for me to gather information to hypothesize about her positions in the classroom, and yet her silence also told a story about her failure in the classroom. I gave this student the pseudonym of Michelle. The second student I chose for her eagerness to participate as much as the teacher would allow and for her failure to turn in assignments, to which her teacher attributed her low grade. The teacher said that her grade in the class was improving because she had started turning homework in. However, I was intrigued that a student could participate in inquiry-based mathematics as much as she did and still have one of the lowest grades in the class. Since she was so vocal, I had enough data to hypothesize about her positions in the classroom as well. Initially I thought a compelling story would be a student who should be succeeding in mathematics but was not, like the second

student that I chose. Unfortunately, this student failed to complete her interviews with me, so I had to drop her from the results section of this study. However, the first student provided a different compelling story about failure.

When I conducted my study, I avoided positioning these students as failures since that was a negative position that might cause students to distrust me. Also, labeling students as such would make them less willing to participate in my study because accepting to participate would mean they were accepting the status of failure. I avoided this negative positioning by telling the student participants that I was intrigued by their behavior in the class and I wanted to know more about their perceptions of the class and mathematics.

Data Collection

In order to gather pertinent data to answer my research questions, I used observations, field notes, memos, student journals, and interviews to triangulate my data and add support to my hypotheses and theory. All of these data collection methods were interrelated and influenced each other. The observations, field notes, memos, and student journals provided me with data that influenced the questions and direction of the interviews. The interviews helped me make sense of what I noticed during observations, recorded in field notes and memos and gathered from student journals. The student journals helped me document the student participant's perspectives of the class. The field notes documented what I noticed during observations. Because of the complexity of human interaction, multiple sources are necessary to glean a better picture of the student-teacher interaction occurring in the classroom (Corbin & Strauss, 1990; Eisenhart, 1988; Romberg & Collins, 2000). Each data collection method provided a different angle for me to strengthen my hypotheses about the positions.

Because I studied the phenomena of positions through the eyes of the students, this was an ethnographic study (Creswell, 1998; Eisenhart, 1988; LeCompte & Preissle, 1993; Merriam, 2002). Since I attempted to interpret my own experience in the student's and teacher's world through my perspective as an outsider, non-interactive methods such as video recordings, field notes, and memos were appropriate. These non-interactive methods captured and recorded the discourse, or language in use, and some of the actions of the participants, better than my attention or memory could. However, these non-interactive methods were insufficient to capture the Discourse occurring. With non-interactive methods, I could only gain partial access to the meanings behind the words, gestures, and actions of the participants. Interactive methods were necessary for me to make sense of what I recorded with the non-interactive methods. Therefore, interactive methods like participant observation, student journals, and interviews allowed me to capture the full Discourse occurring (Eisenhart, 1988; LeCompte & Preissle, 1993). Additionally, I believe that students behave sensibly and that one needs to be embedded in the students' experience in order to make sense of and reconstruct their behavior and thinking, which also calls for interactive methods. With these interactive methods, I attempted to both describe and explain student positioning in an inquiry-based classroom through the students' and teacher perspective (Corbin & Strauss, 1990; Romberg & Collins, 2000). Since I conducted an ethnographic study, ethnographic methods were appropriate and helped me address my research questions.

Non-interactive methods. To capture and record the language in use and some of the gestures and facial expressions that were part of the larger Discourse, I used video recordings. First, I had a microphone on the teacher and a separate mike to use when students worked in smaller groups (Hall, 2000). I had an assistant to run the video camera so I was free to mingle

with the students and take my own field notes. During whole class discussions, the camera was situated toward the back and far side to be less intimidating to students and to avoid recording students without a signed permission form. I had the camera focused on what was happening at the front of the classroom during large class discussion and on a smaller group closest to the camera during smaller group activities. Before I chose my student participants, I focused the camera on the teacher. After I chose my student participants, I focused the camera on them as much as I could. To facilitate their appearance in the range of the video recorder, I asked the teacher to seat the student participants near the camera so I could focus on capturing their words and the other elements of Discourse during whole class and small group work. The video recording was meant to capture the dialogue between the teacher and the students with many of the accompanying actions, like facial expressions and gestures. The meaning behind certain phrases could be clarified through observing the accompanying actions. Because the camera was towards the back and side of the classroom and the lens could only capture a small view of the student participating, I was not able to capture every gesture and facial expression.

While I engaged in observations, I documented my experiences and thoughts through detailed field notes (Marshall & Rossman, 2006) which I later wrote up as memos with my impressions, conjectures, and formulating hypotheses (Miles & Huberman, 1994). I sat on the far side of the room close to the front to take field notes on both student actions and words, particularly of my student participants. These field notes were particularly important since the camera was not able to capture everything due to its location and the location of my student participant. From these notes, I did a brief analysis, in memos, to record my thoughts and impressions and to record my developing hypotheses about the positions that this failing student took on, which was part of answering my research questions.

Interactive methods. In a more interactive approach to observations, I acted as a participant ethnographer as much as possible (Creswell, 1998; Eisenhart, 1988; LeCompte & Preissle, 1993) and thus I was one of the instruments for data collection. Again, only humans can capture Discourse and so I had to have a more involved role with the students. I tried to talk to and interact with students as a friend and confidant to gather information about their Discourse models and the positions. There were two periods of observation, initial and primary, in which I was involved as a participant ethnographer.

Because of the nature of my research questions and of the classroom, I conducted three weeks of initial observations. Through these initial observations and the accompanying field notes, I identified the two students who exhibited the characteristics I was interested in studying further. Through field notes, I documented the students that were resisting the teacher, disruptive to other students, off-task, or disengaged. I also documented students who were communicative and from whom I could glean information about their position and Discourse models. Also, this period of initial observations allowed the students and the teacher to get used to my presence and the presence of the video camera. I needed time to position myself as an advocate who was genuinely interested in the students' experiences. Students needed to feel that I was being genuine because then they were more likely to open up, confide in me, and let me into their world. I tried to do this by talking to students and getting to know them and by talking to them about the mathematics. I tried not to position myself as one who had authority over them because I wanted them to share their experiences with me as they would to a peer and friend. In summary, an initial observation period was very beneficial to my study.

After the period of initial observations, there was a five week period of primary observations. Initially I had planned on four weeks for primary observations, but I wanted to get

two weeks of observations with my student participants directly in front of the camera so I extended my primary observations another week. In five weeks, I was able to identify some of the common and typical positions and storylines in an inquiry-based classroom by looking back at my observations. I used field notes in order to record significant events to talk with my student participant during interviews later. To me, significant episodes occurred when my student participant was actively engaged in the class discussion. She did so by asking questions in the class discussion, challenging other students and the teacher (socially or mathematically), or by struggling with the teacher over an incorrect answer. By making note of these significant episodes, I had a quick reference to these episodes for interviews and later analysis of positions as well as examples to use for my final thesis. Soon after recording my field notes, I wrote them up more formally as memos where I did some initial analysis by making conjectures and forming hypotheses.

In addition to observations and interviews, I had the students keep individual journals about their experiences in this particular classroom. Over the course of my primary observations, I gave my students three journal prompts. Because of Michelle's intense interest in drawing, I first had them draw a picture of their mathematics classroom. This allowed me to glean information about how they viewed their mathematics classroom. For the second question, I had them engage in another creative activity where they wrote me a story about a typical day in their mathematics classroom. Again, I was trying to understand how they perceived the activities and the teacher of this particular classroom. Finally, I asked them to draw me a picture of three students in their class and to tell me about them. Again, this was to help me gather information about their perspective of their peers and to find out who had the most significant influence for them in this class. This also provided insight about what they valued in particular people and

even in the class. Unexpectedly, the teacher also provided me with an assignment from the beginning of the school year where the students had to draw a picture of themselves in a mathematics classroom and to provide information about their experience in mathematics classrooms. I added this to their journals. This also helped me gather information about how they perceived themselves in a mathematics classroom, which also helped me make sense of their positions. To gather more information about what they drew and wrote, I asked them for more details about their journals during interviews.

Outside of the classroom, I conducted three semi-structured interviews (Goos, 2004) with Michelle and four semi-structured interviews with the teacher. The interviews were semi-structured (Zazkis & Hazzan, 1999) so I could ask follow up questions in order to identify positions. Overall, interviews served two purposes: to reconstruct the student and teacher histories in the Discourse and to reconstruct their perceptions of current class events and of each other. In particular, the positions in the teacher's and student's past experience helped me make sense of and confirm the positions they perceived in their current experience.

I interviewed the student and the teacher about their background and history in mathematics education, particularly in the first and second interviews. I wanted to construct Michelle's and Mr. Green's histories through interviews in order to better understand the world through the student's and teacher's perspective. These histories also enabled me to reconstruct their Discourse models and helped me view the world through the eyes of my participants.

Within these interviews to get at the student's histories (personal, mathematics, family, and school) and teacher's histories (personal, school, and mathematics), I also asked them about the perceptions that other people have of them. I asked the student how she thought her friends saw her, how she thought her peers saw her, how she thought her teachers saw her, and how she

thought her mathematics teacher saw her. This helped me to identify how the student felt others were positioning her. Similarly, I asked the teacher how he thought his coworkers and how his students saw him to help me identify how the teacher felt he was perceived by others. Further, I asked them questions about how they viewed school, mathematics, and other students. These questions were to get at what the student and the teacher valued, which would impact which positions they would take on and place other people in. Finally, I asked them about their previous experiences with mathematics education, either as a student or as a teacher. For the student, her experience led her to have a certain view of mathematics education. For the teacher, his experience is what led him to teach inquiry-based mathematics instead of traditional mathematics. This information helped explain why they acted as they did towards the teacher and/or students in an inquiry-based classroom. Identifying how the student and the teacher viewed the mathematics classroom in terms of their history in this environment helped add validity to my hypothesized positions as I considered how well they matched with what they told me about their histories.

By coming to an understanding of their histories early on, I was able to make more sense of their responses and comments to in-class incidents. I incorporated questions about in-class incidents throughout the first three interviews because I wanted to get their perceptions of this particular mathematics class as soon as possible so I could start hypothesizing about positions. These questions about in-class incidents were based on the significant events I recorded in my field notes. For example, I observed and recorded in my field notes that my student participant was passively engaged in the class discussion at times and at other times she was an active participant defending an answer. This seemed especially peculiar because in her interview about her history in mathematics she admitted that mathematics was one of her worst subjects and the

class she enjoyed the least. So, in an interview about the in-class incidents of her varied participation, I was able to ask her about when she chose to actively participate and when she did not. So, asking my participants about in-class incidents allowed me to glean information on how they positioned themselves in the classroom and to ground that in what I gathered about their mathematics education history. In terms of Discourse, interviews helped me identify the values and beliefs they held, which were not necessarily obvious during observations.

During all three student interviews, I also took the time to ask her about what she had written in her journal. For the first journal prompt about the picture of her classroom, I asked her about each person that she drew, their placement, the activities of the classroom captured in the picture, and what she was doing. These interview questions were to get at how she viewed mathematics classrooms, mathematics teaching, and mathematics learning. I used this information to make sense of the positions I was seeing in the classroom. For the second journal prompt about the story, I had her tell me about how much she valued each activity that she mentioned. Because what she valued would affect her positioning, I asked questions to identify what she valued. For the third journal prompt about her classmates, I asked her for more detail about why she chose to draw each person and about their facial expressions. By finding out more about her perceptions of others, I tried to gather how she was positioning herself in relation to other people. For the assignment that Mr. Green provided for me, I asked her about why she drew and wrote what she did. In coming to understand her perspective about herself in mathematics classrooms, I could better hypothesize about her possible positions in the mathematics classroom.

In the last teacher interview, I asked him about his perspective on what I was hypothesizing about the student positions in the classroom. I first asked him to categorize certain

students in terms of their roles in the classroom. Then I shared with him some of what I observed and hypothesized about and asked for his opinion. This was an attempt to do an early member check of my initial analysis. I wanted to make sure that what I was seeing and hypothesizing was in line with what the teacher, a participant of this environment, was also seeing. Therefore, this interview also helped me consider the positioning in the classroom.

I tried to be careful about how I worded questions in order to maintain my position as friend and advocate while also getting my participants to speak freely (Brenner, 2006; Marshall & Rossman, 2006; Zazkis & Hazzan, 1999). I avoided sensitive topics like the student's performance in the class. I also avoided using "why" questions in direct reference to my participants since these types of questions can put the participants on the defensive and sometimes they do not encourage useful answers (LeCompte & Preissle, 1993). Instead, I focused on asking how they viewed a particular situation, how they felt about particular things (like mathematics, school, learning, etc.), what they thought about certain people (teachers or students), and so on. I tried to be aware of how I influenced their responses, like how I responded to their answers (Roulston, deMarrais, & Lewis, 2003). In line with the theory of positioning, during an interview, the participant was trying to figure out how to behave and engage in the interview to give me the answers I was looking for. So, I tried to emphasize to the student that I was trying to tell her story and that I was genuinely interested in what she had to say.

After I mitigated these issues while preparing to interview and during interviews, I expected my participants' thoughts and impressions of the events to give me insight on how they were viewing the positions. The student's and teacher's input helped me to validate or refute my

hypotheses about their positions. Therefore, interviews helped me get at positions from a different perspective, helping me to triangulate my data.

Reliability and validity. With these various data collection methods, I tried to add to the reliability and validity of my study. By nature, reliability has been a challenge for qualitative and ethnographic researchers since they are looking at very specific conditions. In my study, every student's experience was unique and it would be impossible to find two students with exactly the same experiences. However, according to the recommendations of LeCompte and Preissle (1993), I attempted to increase the level of reliability of my study in the following ways. First, I have increased the level of external reliability by carefully describing my position as the researcher, my choice of informants, my social situations and conditions, my theoretical constructs and underlying assumptions, my methods for data collection, and my methods for data analysis. Now that I have finished my study, I have tried to be even more explicit about these factors, which increases the likelihood that another researcher could replicate most of these conditions. Second, I have increased the level of internal reliability, which I address more in the next section, by using verbatim accounts, participants to check my work, peers to examine my work, and mechanically recorded data through video recordings. Even though I can increase the level of reliability of my work, it would be difficult to replicate every condition of my study. However, another similar study would also be able to identify the positions and how they contribute to failure in that particular setting.

Furthermore, my study, as an ethnographic study, has inherent potential for validity (Eisenhart, 1988). With the multitude of data gleaned from these multiple sources and perspectives, I looked for disconfirming evidence to my theories of positions and storylines.

Therefore, observations, field notes, memos, student journals, and interviews provided evidence to support my theories about positions and storylines in an inquiry-based classroom.

Data Analysis

Through my video recordings, memos, student journals, and interviews, I hypothesized about what possible positions were occurring in the inquiry-based classroom that contributed to student failure. From my initial data gathering and observations, I generated hypotheses. Then with subsequent data collection, I tested these initial hypotheses. Therefore, my research questions guided my data analysis.

To facilitate data analysis, I went back to my field notes and transcribed some of the significant episodes. As a review, I considered events significant if my student participant was actively engaged in the class discussion such that she was asking questions, challenging other students and the teacher (mathematically and socially), or struggling with the teacher over an incorrect answer. In these episodes, there was typically a lot of Discourse for me to use to build my hypotheses. As I progressed in my analysis, I went back and watched videos to find other significant episodes that I did not note in my field notes to transcribe. In order to protect the identity of my participants, I gave all the participants pseudonyms. With these transcripts, I had a collection of speech acts to analyze and make sense of based on the other data I collected.

In order to identify the positions in my data, I used the triad of positioning, storylines, and speech acts (Langenhove & Harre, 1999). My data consisted of a collection of speech acts. These speech acts and the accompanying elements of Discourse helped me hypothesize about what the positions were. Since speech acts and other discursive elements like gestures and facial expressions are a part of Discourse, discourse analysis (Gee, 2005; Goos, 2004; Jansen, 2008; Setati, 2005; Truxaw & DeFranco, 2008) was an invaluable tool in interpreting the meaning of

the speech acts and other elements of Discourse for identifying the positions. Discourse analysis also helped me identify the parts of the storylines necessary to identify the accompanying positions. As noted previously, Gee (2005) described language, and thus storylines, as having two functions: to support the performance of social activities and to promote a particular perspective. Attending to how speech acts and other elements of Discourse accomplished these functions helped me identify the positions.

Gee (pp. 11-13) provided me with seven concepts with related questions that helped me analyze the speech acts and other elements of Discourse, but I focused only on activities because they were enough to help me identify the positions. Activities are elements of storylines, and I found that attending to the activities of the storylines was enough for me to identify positions without having to identify the accompanying storyline as well. When I looked at speech acts individually and as a collective group, I asked, "What activity or activities is this piece of language being used to enact (i.e., get others to recognize as going on)?" (p. 11). After answering that question, I could then answer the larger question, what position or positions is this piece of language being used to enact (i.e., get others to recognize as a player)? What and how something was said helped me hypothesize about what the possible positions exist in the Discourse.

When given a set of speech acts, I coded them to identify possible positions for my student and teacher participant. With each speech act, I identified the activities, perspectives, and purposes expressed. Then I considered if there was another way it could have been said (Gee, 2005). This helped to reveal how the speaker was positioning him or herself as well as how the speaker was positioning another. There was no way I could identify all possibilities, but this was a useful exercise. After considering other ways that it could have been said, I identified

a possible position that the participant was taking on. The positions were my own constructions that I used in order to help me make sense of my data. I tried to capture the intent of the participants in the label of the position. Further, interviews and observations helped me consider the Discourse models of the student and teacher, which also helped me to check the positions I identified through speech acts. I did not actually construct the Discourse models for the participants, but I used their histories and their reported perceptions of the mathematics class and other participants to compare the positions to. I tried to check to see if the position was one that the participant was likely to actually take on based on their self reports. This way I made sure the positions and speech acts I identified were in fact close to the student's and teacher's perceptions of positions in inquiry-based mathematics classrooms.

By using these principles to guide my analysis, I further coded the speech acts and refined my hypotheses about positions, using open, selective, and axial coding procedures as put forth by Corbin and Strauss (1990). First, during open coding, I looked for similarities and differences in my positions (by comparing the speech acts and activities) in order to merge some positions as well as to refine each position. With each position, I considered how it was distinct from other positions by examining the distinctions between the speech acts and activities. When certain speech acts and activities seemed to be trying to accomplish a similar purpose, I combined them into a single position. Similarly, if the speech acts and activities seemed to be trying to accomplish different purposes, then I created separate positions. Then, with selective coding, I worked to fill each position with descriptive detail, which is important to the replicability of my study. I looked at multiple examples of speech acts and activities under a single position to add to this descriptive detail. Finally, in axial coding, I looked across my positions to draw connections between them. I looked for common themes and relationships

between positions that could be contributing to student failure. Therefore, the process of open, selective, and axial coding helped me refine and fortify my hypotheses about positions.

I started my data analysis during data collection so I could used newer data to check the hypotheses I had created (Miles & Huberman, 1994). I identified positions as I collected my data to insure that I was getting the data I needed to answer my research questions. As I reflected on the data I was collecting and how it related to my research questions, I was able to modify my data collection methods as needed. In this way, I was doing a form of the constant comparative method (Corbin & Strauss, 1990; Eisenhart, 1988; Gee, 2005; Goos, 2004; Jansen, 2008; Setati, 2005; Truxaw & DeFranco, 2008). I came up with hypotheses about the positions, and then tested these hypotheses against the new data I collected. This helped me find common positions that were occurring and added support to my developing hypotheses. If the speech acts and activities I identified fit with certain positions, then my hypothesized position had more support. If they did not fit, then I worked on refining my positions until it did. By doing so, I was able to refine my positions, look at how they were related, look for possible contradictions in new data, find explanations for variations, and eliminate positions that were not well supported by the data.

Additionally, my research advisor and peers looked over some of my data and checked the plausibility of my hypotheses about the possible positions (Miles & Huberman, 1994). Due to my lack of time, I was unable to take my refined positions back to the teacher and the student of my study in order to have them member check and verify or refute my analysis (Eisenhart, 1988). However, the teacher will receive a copy of my finished thesis per request as an effort to improve his teaching. Having outsiders to my study along with the participants checking my

results adds to the validity of my findings. Overall, I worked with my data analysis to make sure it was convincing and believable.

Chapter 4: Positions and the Negotiation of Positions

Common Positions

In the following chapter, I introduce and describe the common positions of the student and the teacher, and then I talk about the complex negotiation of positions that occurs between the teacher and the student and how it could lead to failure. Student failure in inquiry-based classrooms means more than just a low letter grade in the class or the student being off task. It also means that the student is engaged in traditional mathematical learning, such as seeking for direct instruction and validation. In this way, students are not problem solving and reasoning about the mathematics independent of an authority source. So, the student fails when he or she does not engage in inquiry-based mathematics. As for the positions I talk about here, this is not a comprehensive list of positions, and since positioning is a result of complex social interaction, it would be impossible to enumerate every possible position that a single individual takes on. In this chapter, I talk about six common student positions and six common teacher positions (see Table 1).

Table 1

List of Positions

Student Positions	Teacher Positions
1. Uncertain Student	1. Introducer
2. Dependent Student	2. Orchestrator
3. Assertive Peer	3. Behavior Manager
4. Mathematical Authority	4. Pseudo-Orchestrator
5. Distracted Student	5. Administrator
6. Engaged Listener	6. Teller

These common positions are particularly significant because they seem to be the participants' preferred positions and seem to influence the other participant's preferred positions. Further, these common positions are directly related to student learning. In terms of discussing

and describing individual positions, positions are defined by the specific activities and speech acts that the participant engages in, so each position is first described in terms of activities. With each position, I describe what it means for the participant to take on that position and how he or she positions other people while he or she is in that position. I then provide examples of the speech acts from transcripts associated with the position. In the second half of the chapter, I describe how the positions interact and influence the participants of the discourse.

Positions associated with the student. Table 2 is a summary of the positions associated with the student in terms of activities.

Table 2

Descriptions of Student Positions

Description	Activities
Uncertain student	Seeking validation from teacher
Dependent student	Seeking instruction, explanation, and answers
Assertive peer	Making social judgments
Mathematical authority	Voicing and defending answers publicly
Distracted student	Disengaging from the class
Engaged listener	Listening attentively without direct involvement

Uncertain student. When Michelle engaged in the activity of seeking mathematical validation from the teacher in this inquiry-based classroom, she would take on the position of uncertain student. As an uncertain student, she had decided on an answer to a particular problem but was uncertain about her answer and thus sought for validation from the teacher and the teacher only. Even when the teacher was not paying attention to her, she would still wait for his validation. However, she was not shy about being uncertain in front of her peers, which meant that she would present even if she was not certain about her answer. From interviews, she told me that she liked to present to show the teacher and the other students that she "gets it." With

the uncertain student position, she was showing that she understands some of the mathematics. While in this position, she positioned the teacher as a validator and positioned her peers as witnesses of her performance.

The following is an excerpt from one of the transcripts that shows a good example of Michelle taking on the position of uncertain student. In this segment, Michelle volunteered to present her line of best fit even though she was not sure it was right.

Mr. Green: Um, is there somebody that would show the line of best fit for . . . for this first one? [Michelle raises her hand] Somebody got a good one? Michelle you want to bring yours up?

Michelle: [Inaudible] Is that right? [Shows Mr. Green her paper]

Mr. Green: [Pauses] Tell us about it. Go up right up there and like --

Michelle: Ok, well. This is my line of best fit, and . . .

Mr. Green: James, shh.

Michelle: [Inaudible] . . . hit the most dots. So, I just . . . I started at this dot [inaudible], and then I went like that. And then I put it on this side because . . . um . . . you put it on the side that has the more dots because you want it to be closest. Right? [Looking at teacher]

With this segment, you can see Michelle had an answer but was seeking for validation from the teacher in front of her peers when she asked "right" twice while her facing the teacher when she said it.

Dependent student. When Michelle engaged in the activity of asking the teacher or other students for mathematical instruction and explanation, she would take on the position of dependent student. While in the position of dependent student, Michelle was not seeking for validation of an answer she had worked on (as with uncertain student). At this point, she had given up suggesting answers. As a dependent student, Michelle was seeking for the teacher or another student to tell her the answer, to explain how to get the answer, or to explain why that

was the answer. She positioned the teacher as a smart person with authority. As for her peers, she positioned them in various ways. From interviews, Michelle considered a few of the smart students to be indiscernible as she felt she could not understand their utterances, so she positioned these students as indiscernible. With other smart students, she valued them for their explanations but worried that they might get annoyed with her for asking for help, so she positioned these students as helpful. For other students, she positioned them as unhelpful because they were independent workers and would not provide her with explanations. There were yet other students in the class that she positioned as equals because they were at her level of understanding.

In the data, there were various examples of this position. If she was presenting, she frequently expressed confusion by saying, "I don't know" and "I am getting confused." These expressions were like pleas for help. She first turned to the teacher and if he did not help she would then turn to the classmates for instruction. If she was working at her desk, she would usually turn to the student in front of her because he was "smart" and could explain the mathematics clearly.

Assertive peer. When Michelle engaged in the activity of making social judgments on her peers' comments, she would take on the position of assertive peer. She would tease and critique her peers based on their behavior and would critique some of their non-mathematical comments. Her friends even described her as brutally honest. From an interview, Michelle considered herself as popular but, "If the popular people were like a body . . . then I would be the tail," which may be why she felt comfortable making social judgments on her peers. As for the teacher, she occasionally passed social judgment on him, like when she thought he was being unfair. At other times, Michelle (as the assertive peer) tried to create a more informal

relationship with the teacher to try to create solidarity, which is addressed more in the next paragraph. In terms of positioning others, there seemed to be two categories: insiders and outsiders. The insiders were the ones who either took part in her making social judgments or were subjects to her social judgments. She would position some insiders as obnoxious and some as entertaining. However, she only passed her social judgment with those who engaged and challenged her at a social level. Smart and quiet students, not part of the popular crowd, were usually not subjected to her social judgments, making these students outsiders. As for the teacher, he was a differentiated insider because Michelle passed social judgments on him in a different way than she did with her peers.

The following are examples of her being an assertive peer with her classmates and with the teacher. For the insiders, she gave commands like, "Can't hear you" or she would taunt student who challenged her like, "Oh my gosh, Mike" or she would classify a peer's non-mathematical comment as "dumb." For outsiders, she usually did not pass social judgment and would occasionally protect them from the social judgments of her peers. One time, when her peers were teasing a student for being wrong because he was usually right and vocal about being right, she told them to stop. All of these examples show that she felt like she could make judgments on her peer's behavior and comments during class. For the teacher, during one class, he claimed he did not call on her because her hand was not raised and she challenged him by telling him that she had been raising her hand, which shows her passing social judgment on him. Even though she would pass social judgment when she felt the teacher was being unfair, she did not pass social judgment on his other comments in the class (like his explanation, instructions, disciplinary comments, verbal fillers, and incomplete statements). Another time, she called out "Green" to get his attention. In an interview, she told me that she was not trying to be

disrespectful but was trying to make him more like a collaborater where they work "all together as one" and he was not just a "dictator." From Michelle's actions, she did not seem to really position him as a collaborator because he was always a source of validation for her. However, this does indicate an interesting relationship that she was trying to establish. By calling him "Green," she was removing his title. Formal language can create distance emotionally and socially (Gee, 2005, p. 39). Informal language, on the other hand, creates closeness emotionally and socially. Michelle may have used informal language to create solidarity so that the teacher would judge her on more than her cognitive abilities alone. Furthermore, she told me in the same interview that her peers would probably want to call him that too because "it's kind of cool." Therefore, she also called him "Green" to build social status with her peers.

Mathematical authority. While in the position of mathematical authority, Michelle engaged in the activity of verbally demonstrating confidence in mathematics. She usually did this after she had received validation from multiple peers and/or from the teacher (directly or indirectly). Michelle would voice and even defend her answer as well as challenge other students' answers. Even when her answer was challenged by another student or the teacher, she would continue to be confident and defend her answer until the teacher explicitly told her it was wrong. During interviews, Michelle frequently talked about mathematics in terms of "getting it," which referred to knowing the procedure or definition and being able to apply it correctly. Many of her utterances during discussions had to do with defending answers based on how well they fit with what she thought the definition was. She told me in an interview that she would only contribute to discussions if she felt that she "gets it" and there were some people who did not "get it." When she did "get it" and there were students who did not, she would take on the position of mathematical authority. Also, the mathematical authority position had some heavy

social elements that were similar to those in the assertive peer position. Many times Michelle was very competitive with other students as a mathematical authority. In this case, she sometimes passed social judgment through defending mathematics. Because of the social aspect, Michelle was performing for the insiders (from her assertive peer position). When some of these insiders disagreed with her answer, Michelle would position these students as ones in need of correction. For the insiders and outsiders that did not "get it," she would position them as in need of instruction. For the people (including the teacher) who supported her, she positioned them as validators.

The following is an excerpt from one the transcripts that illustrates the mathematical authority. During part of one of Michelle's presentations, Michelle claimed the slope of this line of best fit was two over three based on the points (1, 100) and (3, 300) on the line (see Figure 1). She reasoned that the distance from one to three was three units by counting the lines (the run), and the distance from 100 to 300 was two squares and thus two units (the rise). She decided to count squares to determine slope based on what a substitute had told her the previous class period. This created problems for the teacher since many students firmly believed the rise of the slope was two. One student recognized the error in the rise and asked her about it, which is where this segment starts.

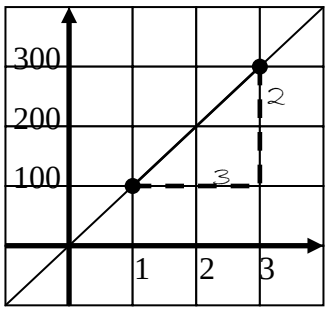

Figure 2. Michelle's graph of the line of best fit.

Mike:	Wait, 2 over 3, what do you mean?
Mr. Green:	[Teacher points at Mike] There's some question . . .
Michelle:	2 [points to rise of her triangle] and that 3 [points to run of her triangle]. Look, 1, 2, 3.
Mike:	Yeah, but is that really going up 2? Or is it going up like 200?
Student:	No, like, it's going up like 2 squares.
Michelle:	2 squares.
Mike:	Yeah, but each square isn't worth one. Is it?
Student:	Yeah.
Mike:	[Quietly] It's worth like 100.
Michelle:	Can't hear you!
Mike:	Each square is worth 100!
Michelle:	Oh.
Jake:	Well, . . . it's still right.
Michelle:	So, [inaudible]
Student:	No, I think she's right.
Student:	She's right.
Michelle:	Ah-huh, it's not the number it's the squares. [points at Mike with the smart board marker]

From Michelle's first statement where she pointed to her answer, Michelle was confident in her answer and so took on the position of mathematical authority. When she said "can't hear you," she stepped into the assertive peer position which shows how closely the assertive peer is related to this position. As other students validated her answer, she grew more confident as a mathematical authority to the point that she taunted Mike for challenging her by saying, "it's not the number it's the squares."

Distracted student. At certain times during class, Michelle would engage in the activity of being off-task and disengaged from the class and thus took on the position of distracted student. I identified periods when she was distracted because she would start doodling and socializing with certain people around her instead of listening attentively to the teacher and contributing to the discussions. She told me that when she was engaged in doodling, she was still listening to what was happening in the class. She also told me that there were times during class when she was distracted because the class was boring. When the teacher would excessively review something that the majority of students already "got," she said the class was boring. When she took on the position of distracted student, she positioned the teacher and some students as boring. When she engaged in the activity of socializing, she positioned other students as distracting or entertaining.

The following is an example of her in the distracted student position. While the class was reviewing warm-up problems, Michelle was engaged in doodling. After one student had shared her answer, the teacher checked for understanding, which is where this segment begins.

Mr. Green: Are we ok with that? Very nice.

Sunny: No, I didn't hear what she said.

Mr. Green: There a question?

Michelle: [Sketching and does not look up] She couldn't hear.

From her doodling and failure to look at the teacher when she addressed him, she was taking on the position of distracted student. This segment demonstrates that even as a distracted student she could still be listening to what was going on in the class.

Engaged listener. When Michelle was attentively listening to the teacher and taking notes without directly participating in the discussion, she was taking on the position of engaged listener. This position was especially marked by Michelle's silence. However, she was not

doodling or talking to other people. Her attention was strictly on the teacher and what he was saying, and she even occasionally made gestures to indicate that she was following the discussion. At times, she seemed to be listening for the answer or information on how to get the answer. At other times, there was evidence to indicate that she was reasoning about the correctness of what was being said instead of just looking for an answer. While in this position, she positioned the teacher as a mathematical authority in the classroom and positioned some students as smart.

For example, the teacher had talked about how a vertical line was a counterexample to a students' definition of line of best fit. The following segment shows Mr. Green addressing this same issue for a second time and shows Michelle's reaction.

> So, you cut all of the dots in half. Now be careful about that because when I went like this you said no [Michelle draws vertical line in the air with her hand to show the line teacher was going to draw], I got half here and half there.

This segment shows that Michelle had been following along with the discussion and remembered the counterexample of the vertical line, which she indicated with her hand. This also showed Michelle reasoning about the student's answer. Therefore, she took on the position of engaged listener during this discussion.

Positions associated with the teacher. Table 3 is a summary of the positions associated with the teacher in terms of activities.

Table 3

Descriptions of Teacher Positions

Description	Activities
Introducer	Introducing tasks and activities Telling students what they are going to do and why
Orchestrator	Directing class discussions and activities Engaging large number of students in commenting, contributing, and asking questions

	Summarizing key ideas
Behavior Manager	Correcting students behavior verbally
Pseudo-Orchestrator	Making more direct comments Asking closed and leading questions
Administrator	Taking care of administrative teacher responsibilities
Teller	Providing students with direct answers

Introducer. In an effort to enact inquiry-based instruction, Mr. Green engaged in the activity of introducing the activities and tasks of the day as well as topics of discussions. While engaged in this activity, he took on the position of introducer. As an introducer, he usually started a task or discussion by informing students of what they were going to be talking about and why. Also he usually established guidelines for how students should engage and behave (like sharing their ideas, presenting at the board, or recording data to graph) in the task or discussion. Introducing was usually not a question but a statement such as "your job is to guess how old you think they are." Occasionally while in the position of introducer, he explained the purpose or objective of the task or discussion. As Mr. Green introduced an activity, he expected students to listen silently but attentively. If students were talking, he told them to stop, and if students were not paying attention to the introduction, he did not repeat himself. This position would usually precede the orchestrator position. While in this position, Mr. Green positioned students as inductees to the task or discussion.

As an example of the introducer position, Mr. Green introduced the discussion of finding the equation for the line of best fit by saying:

> Alright, as I went around, I saw some good lines of best fit, but I think we need to probably discuss the equations because I see that some people don't have theirs. We need to make sure that we can do that.

In this excerpt of the introduction, you can see Mr. Green engaged in the position of introducer. He told students that they were going to be talking about the line of best fit.

He also informed them that they were expected to "discuss" the line of best fit. He justified the discussion by telling the students that not everyone had their line of best fit. Finally, he alluded to a larger purpose or objective of the class as a whole, which was being able to write their equations for the line of best fit.

Orchestrator. Since discussions (in small groups or as a whole class) are key features in inquiry-based instruction, it is necessary that the teacher engage in the activity of directing the discussions so that student ideas and contributions to the discussion are respected, clearly expressed, and focused on the important mathematical ideas. While engaged in such an activity, Mr. Green took on the position of orchestrator. Mr. Green tried to encourage a large number of students (usually four or more) to participate in the discussion by asking questions and making comments. Also, Mr. Green asked clarifying questions to encourage students to express their ideas clearly and to consider certain cases, he summarized conclusions and key ideas of the discussion, and he avoided giving direct answers to promote students to think for themselves. Because of how Mr. Green wanted students involved in the discussion, he positioned them in ways typically allotted to teachers only, like contributors, endorsers, questioners, and challengers.

While in the position of orchestrator, the teacher would ask questions and make comments like "Michelle, you want to bring yours up," "What do you guys think," and "Why would the actual age be a good independent (variable)?" These examples show the teacher turning the discussion over to students for them to contribute. Also, these examples show the teacher giving the discussion a mathematical direction for students to follow in the discussion.

Behavior manager. At times, teachers feel pressed to engage in the activity of managing student behavior by verbally correcting the disorderly students. While engaged in this activity,

Mr. Green took on the position of behavior manager. In interviews, he admitted feeling pressed by student behavior. He wanted to allow students to talk to foster an environment for students to express their ideas, but at the same time, he felt students took advantage of the ability to talk and engage in off-task behavior, like socializing. When students were talking over each other, were not raising their hands, were making irrelevant comments (like to entertain), and were socializing, Mr. Green tried to correct their behavior by saying the name of the disruptive student or by instructing students on correct behavior. When he felt the students were breaking important classroom social norms, Mr. Green took on the position of behavior manager. While in the position of behavior manager, Mr. Green positioned these talkative and norm-breaking students as disruptive.

For instance, "Jason, shhh," "Jake, sit down," and "one at a time" are all examples of Mr. Green being a behavior manager. Students were usually quick to obey such commands without complaint. The teacher tried to fix these particular instances of behavior to promote behavior that supported the mathematical activities of the class. As a behavior manager, Mr. Green was trying to clear a space for mathematics instruction to occur.

Pseudo-orchestrator. Sometimes students make comments that are not productive to the objectives the teacher has for the discussions. Mr. Green felt responsible to make discussions productive and relevant to the mathematical objectives. When the students were not moving towards the right conclusions, Mr. Green felt pressed to engage in the activity of making more direct comments or asking more closed and leading questions (to direct students to certain answers and conclusions) while still not directly telling students answers. While engaged in such an activity, Mr. Green would slide into a pseudo-orchestrator position. This position was characterized by the teacher trying to tell students something about mathematics while still trying

to maintain inquiry-based instruction. In the orchestrator position, the teacher tried to position students as contributors, endorsers, questioners, and challengers. Since Mr. Green was trying to press students towards certain conclusions, he was the one challenging and asking questions so he did not allow them to take on these positions. He still allowed them to contribute and endorse, but by asking closed questions and making more direct comments, Mr. Green restricted the answers that students could contribute and endorse. So, Mr. Green positioned students as pseudo-contributors and pseudo-endorsers because he valued the direct answers coming from students but tried to get students to contribute and endorse particular answers.

As an example of the pseudo-orchestrator, the students were debating over if the rise of the slope was the number of squares (two) or if you had to account for the scale of the squares (making the rise 200). At first, students came to the wrong conclusion of two, but after some teacher intervention, the students were stuck arguing over the two different answers again and neither side was willing to yield. After asking a series of closed questions to lead students to the right answer, students were still undecided. To settle the debate, the teacher stepped in with the following question:

Mr. Green:	Is there a difference between 2 and 200?
Mike:	Huge difference.
Mr. Green:	If we are talking about the total amount of money you earned, is there a difference between a 2 and 200?
Students:	Yeah.
Mr. Green:	So is it going to matter?
Mike:	Yes!
Student:	Yeah.
Mike:	It's 200.

In this segment, you can see the teacher taking on the position of pseudo-orchestrator. Each question that he asked was pushing students to one particular answer. The teacher was trying to show students that the answer was 200 without coming out and directly saying the answer. He tried to use students to endorse this answer as well to further convince them of what was right.

Administrator. At times in the classroom, the teacher was called away from tasks and discussions to take care of administrative issues and engage in activities such as handing out homework, explaining non-mathematical parts of assignments (like what he expects), answering phone calls from the office, taking care of students leaving early, and helping students with technology issues. While engaged in these activities, Mr. Green positioned himself as an administrator. This position dealt with the non-mathematical activities that a teacher must engage in to ensure that his classroom is functioning as expected. Mr. Green positioned these students (and office personnel) as information seekers.

For example, while a student was preparing to leave class early, she asked "Is it just this one?" to verify the homework she needed to grab on her way out the door and he responded with "Yep." During the same class period, the office called Mr. Green about the student who left, and he told them "She's gone." These questions required the teacher to take on the administrator position in the classroom. In a way, he was providing the information that only he has access to about his class to the students and the office.

Teller. Because of the difficulties of teaching inquiry-based instruction, the teacher was sometimes pressed to engage in the activity of providing the students with direct mathematics instruction such as telling students what the answer or conclusion should be, telling the students what the important idea was, telling students about mathematical norms, showing students a solution method, or validating answers. Typically, Mr. Green would try to engage students as an

orchestrator or pseudo-orchestrator first. If these positions were insufficient in getting students to the correct or important mathematical ideas in a timely fashion, then he would engage in these telling activities and resort to the position of teller. The teacher would take on the teller position often towards the end of the discussions. When he did this, Mr. Green positioned his students as right, wrong, or in need of direct instruction.

For example, to settle the debate shown in the mathematical authority and pseudo-orchestrator positions, the teacher finally took on the teller positions to inform students that, "they wouldn't have numbered this by 100's if they wanted us to go by 1's." During another class period, there was a short debate among students over whether a graph was a scatter plot or not, and the teacher settled the debate by saying, "yeah, it's a scatter plot." Both of these are examples of telling. In the first, the teacher was telling students about mathematical norms and conventions. In the second, the teacher was validating a student's answer. So, the teacher was engaged as a teller in both cases.

Negotiation of Positions

Now that the positions have been described, I discuss how these positions interact with each other and influence the positions that other participants take on. In this introductory section on the negotiation of positions, I first discuss the patterns I noticed in the student and teacher positions over the course of the semester. Second, I justify talking about the negotiation of positions with just one class period and I briefly describe the class period. Third, I outline how I am going to talk about the negotiation of positions that occur in that class period.

Over the course of the semester, I noticed that the student's types of positions varied more widely than did the teacher's positions. At the beginning of the semester, Michelle took on positions (like the uncertain student and dependent student) that supported traditional instruction

as she sought validation and instruction. Even when she presented, she would take on positions that supported traditional instruction. As the teacher complemented her on what she did right or well, her confidence grew in her work. At one point in the class, the two students who had primarily taken on positions like the mathematical authority position were no longer there, so Michelle and another student (Mike) quickly replaced them. By the end of the semester, she was moving in and out of the mathematical authority position.

As for the teacher, his choice of positions over the course of the semester depended more on the type of curriculum he was using. Mr. Green used an inquiry-based curriculum at the beginning and end of the semester, and so during these units, he took on positions that supported inquiry-based instruction but he also slipped back in positions that supported traditional instruction. During the middle of the semester, Mr. Green used a traditional curriculum to reinforce the ideas of solving equations, and so he took on positions that supported traditional instruction, but tried to involve students more in the mathematics, like in asking for multiple solution strategies. From my observations, Mr. Green seemed more comfortable with traditional instruction. He may have been more comfortable because either students (and other factors) did not press him as much or he did not allow students to press him into other positions as much as with his inquiry-based instruction. His comfort with traditional instruction may be why he slipped back into positions that supported traditional instruction while using an inquiry-based curriculum. From interviews, he believed that inquiry-based instruction promotes more understanding than traditional instruction, so he did try to implement and maintain inquiry-based instruction.

In order to talk about the negotiation of positions that existed in this classroom, I focus my discussion on one class period because it is representative of the positions that expressed

themselves throughout the course of the semester. Since this class period occurred towards the end of the semester, Michelle took on relatively new positions to her like mathematical authority and would slip back into relatively familiar positions that she was accustomed to like dependent student. In particular, Michelle took both very active roles in this class period by participating in discussions and very passive roles in this class by disengaging. Because the teacher was using an inquiry-based curriculum for this class period, he took on positions that ranged from orchestrator to teller. Hence, during this single class period, there was a wide range of positions present. Furthermore, this class period (1) exemplifies the conflict that exists between teachers and students that press them in and out of positions, (2) highlights the student's dependence on different forms of validation for her mathematical performance, and (3) maps the deterioration of positions (as participants are pressed into less desirable positions) and the reclamation of more desirable positions. An analysis of this single class period shows why students could potentially be failing in inquiry-based instruction and why teachers may struggle to maintain inquiry-based instruction.

To illustrate the negotiation that occurs in this class period, I follow the following pattern in my explanation: I provide part of the transcript, I identify the existing positions, I discuss how the positions are being negotiated, and I discuss the implications of the negotiation. The pieces of the transcript that I provide are presented in chronological order to illustrate the negotiation of positions across a single class period. At the end of this chapter, I show why positions are so important to student learning in inquiry-based instruction.

Episode 1: Introduction of a discussion and the beginning of trouble. In this episode, the teacher started a discussion about how to accurately describe the line of best fit. This particular episode outlines the basic pattern of interaction between the teacher and the students

for most of the discussion. Mr. Green elicited a student response, a student provided an inadequate description for the line of best fit, the teacher challenged their description with a counter example, and the teacher reiterated what he wanted students to do. Most of the time, the teacher challenged the students' descriptions because they were incomplete rather than incorrect. Also, this discussion was the source of conflict for Michelle later on. In this episode, the teacher took on the introducer, behavior manager, orchestrator, and pseudo-orchestrator positions while Michelle took on the engaged listener position.

Mr. Green:	Alright, then there's another question there about what is the, ah [students talking] . . . are we alright? What is the line of, ah, best fit? There's a question about that. So, what is it? [Michelle listening and taking notes on her warm-up]
Students:	[Mumbles] Graph.
Mr. Green:	What is the line of best fit? Like if it says . . . explain that . . . line of best fit. [Steven raises his hand and teacher points to him, but Mike answers at the same time] It's Steven's turn.
Student:	Good job, Chandler.
Steven:	[Looks at Chandler briefly] It's line that . . . it's a . . . it's . . . it's guessing where the line would be if it was linear.
Mr. Green:	Alright, so, I have a scatter plot with some dots. [Draw axes and lots of dots with a negative correlation] I collected my information and that's what I got. So, I am just guessing where the line would be, so I am guessing it's about like that [Draws a line in the dots with a positive slope]. Is that my line of best fit?
Students:	No!
Mr. Green:	But it was a guess!
Students:	[Lots of talking]
Brad:	Can I show? [Raises his hand]
Mr. Green:	Ok, so give me a more. I think Steven is headed in the right direction, but I want you to *describe* it without showing me.

In terms of negotiating position, Mr. Green followed this pattern of negotiation through most of the discussion about describing the line of best fit. To set up this discussion, the teacher started by introducing the discussion by taking on the introducer position. While introducing, the students pressed the teacher out of the introducer position into the behavior manager position by talking out of turn. Mr. Green probably wanted to make sure that each student's contribution was heard, so he took on the behavior manager position to make sure this would happen. Mr. Green then took on the orchestrator position by opening the discussion for student contributions by asking, "what is it." When the students failed to raise their hands to answer him, the teacher stayed in the orchestrator positions and felt pressed to reiterate what he wanted students to do. Steven supported the teacher as an orchestrator by raising his hand, but Mike's outburst pushed Mr. Green from an orchestrating position to a behavior manager position because Mike was not allowing Mr. Green to be the orchestrator. Because Steven's description was lacking in descriptive details, the answer pressed the teacher to step out of his orchestrating position. Instead of telling Steven what was lacking from his description, Mr. Green challenged Steven's description with an example of a line that fit Steven's description that was not the line of best fit. By challenging Steven, Mr. Green was indirectly telling him that his answer was not completely correct, so Mr. Green took on the position of pseudo-orchestrator. Even the leading question "is that my line of best fit" indicated the teacher was taking on this position. When students agreed that the teacher's example was not a line of best fit, Mr. Green took on the orchestrator position again to try to press students into giving him a more detailed description of the line of best fit. Throughout this episode, Michelle's fixed attention on the teacher showed that she was taking on the position of engaged listener.

As I said, this was the pattern of negotiation for this particular discussion about what constituted a line of best fit. As the students continued to give him inadequate descriptions of the line of best fit, the teacher felt more and more pressure to step into the teller position in order to provide students with a clear, detailed, and correct description of the line of best fit. However, based on the large number of students Mr. Green allowed to contribute, Mr. Green may have been trying hard not to slip into the teller position by waiting for a student to provide an adequate description of the line of best fit. As for Michelle, she might have seen the teacher's challenges as an indication that the descriptions were wrong instead of incomplete. She watched as student after student failed to give the teacher an acceptable answer. To Michelle, there may have been students that did not "get it." Since no student shared a description close to hers and hence the teacher had not challenged her answer, her confidence in her answer could have been increasing. Therefore, the pressure on the teacher and student continued to build towards a larger conflict.

Episode 2: Michelle's conception of the line of best fit. In this episode, Michelle felt compelled to share her description of the line of best fit. Before this episode, with each new student who joined the discussion, the teacher had a new description of the line of best fit. The students were not building on each other's contribution but approaching the description in different ways in the hopes that the teacher would find their description satisfactory. Because no description had been accepted yet, the students likely saw this as an indication that the correct description had not been given yet. Michelle told me in an interview that she only contributes to discussions when she feels like there are people who do not "get it." Since there were people who apparently did not "get" the line of best fit, Michelle contributed her description of the line of best fit, which is where this episode begins. However, her description indicated some misunderstanding of the line of best fit. From another statement of hers during this class, the line

of best fit to her meant that the line fit (or included) as many points as possible. As the teacher tried to correct this misunderstanding, the discussion was interrupted by a phone call from the office. The office was looking for a student who left his class early for a basketball game. As a result of the phone call, the misunderstanding was not fully addressed and Michelle was still convinced that she was right, which is where this episode ends. In this episode, Michelle took on the position of mathematical authority while the teacher took on the positions of orchestrator and administrator.

Michelle:	[With hand raised] I think I . . .
Mr. Green:	[Points to Michelle] Michelle.
Michelle:	. . . wanna take a crack at it . . . Ok, it's a line on the scatter plot that's linear and connects to the most dots as possible.
Mr. Green:	Ok, so if I can get it to hit . . . like if I can get hit a whole bunch right here.
Student:	No.
Mr. Green:	Would that be a line of best fit? [Draws a line to connect a lot of his dots] I hit one, two, three, four . . . That's probably . . .
Mike:	Cut it in half.
Michelle:	No, the most dots as possible.
Mr. Green:	The most as possible. Is it possible that I could hit a whole bunch of dots and still not really be a line of best fit?
Student:	Yes.
Michelle:	But, linear.
Mr. Green:	Oh?
Michelle:	Mine makes sense.
Mr. Green:	[On the phone] Hello?
Students:	[Talking about their descriptions]

Mr. Green: [Still on the phone] She's gone.

Brad: She's gone.

Mr. Green: She's gone. Jefferson. [Michelle puts her head down on her desk]

During this negotiation, the pressure on the teacher and the student was apparent as well as the student's dependence on validation. By raising her hand, Michelle helped Mr. Green maintain his orchestrating position. The confidence evident in her description showed that she took on the mathematical authority position. Because her description demonstrated misconceptions about the line of best fit instead of incompleteness, the teacher felt pressed to step out of his orchestrating position to correct her. In this case, he was pressed into challenging her by providing a counterexample, which was the same approach he took to incomplete answers. By so doing, the teacher took on the pseudo-orchestrator because he was indirectly telling her that her answer was wrong. Even though Mr. Green tried to press her out of the mathematical authority position, Michelle maintained her position by defending it against the challenges she received from Mike and the teacher. The competition between her and Mike to be the mathematical authority was evident as he claimed the line of best fit "cut it in half" and she countered "No, the most dots as possible." Again, Mr. Green tried to press her out of the mathematical authority position by taking on the pseudo-orchestrator position and asking "is it possible." Because he drew the line free hand, it did not appear to be completely linear. Between this and the teacher's challenge of "the most dots as possible," Michelle may have thought Mr. Green's challenge was suggesting a curve, not a line. To clarify her description, she reiterated that the line had to be "linear." Her additional comment "mine makes sense" indicated that she believed her concept of the line of best fit was correct even if it was not detailed enough, like with the other students' descriptions. Before Mr. Green could try to press her out of the mathematical authority again, Mr. Green was pressed to step out of the pseudo-orchestrator

position and into the administrator position when the phone rang. After the phone call, Michelle's conception was dropped completely as Mr. Green took on the position of orchestrator again and called on another student who provided his own description of the line of best fit, separate from Michelle's description. Michelle showed frustration in the teacher not accepting her answer by putting her head down on the desk. Therefore, by the end of this episode, Michelle was still in the mathematical authority position and convinced that she was right.

In this episode, the pressure continued to build. The teacher felt pressure to slip into the telling position to correct the student's misconception since the counterexample was not enough. The student felt pressure to slip out of the mathematical authority position because the teacher did not validate her description. However, the teacher maintained his orchestrator position and Michelle maintained her position of mathematical authority. Neither yielded which was in part due to the interruption of the phone call. Because of the phone call, Michelle's misconception was overlooked as the teacher attempted to restart the discussion. Hence, Michelle was still convinced that her conception of the line of best fit was right since the teacher had not told her she was wrong. In this particular case especially, the failure to fully address the student's misconception set the stage for a larger conflict later between this student and the teacher.

Episode 3: Validation trouble. The following episode is the final piece before the major conflict that has been building ensues. In this episode, Mr. Green invited Lydia to come draw the line of best fit based on the discussion. Instead, as shown in the transcript below, she gave yet another description of the line of best fit that was very similar to Michelle's previous description. Lydia was a particularly shy student in the class, but she also was good at mathematics. In this case, the teacher validated her description. During this whole discussion, the teacher had been trying to get students to build off of each other's descriptions to create a

detailed description of the line of best fit (which is why he kept repeating the instructions for the discussion) but the students had not done this. Even though Lydia's description was not completely correct, it did combine what other students had said about the line of best fit. Mr. Green was validating the inclusiveness of the description and not necessarily the absolute correctness, but Michelle likely saw it as the teacher validating Lydia for her correctness. As a result, Michelle was agitated with Mr. Green for challenging her and for validating Lydia's description. Also, Michelle was even more convinced of the correctness of her conception because Lydia's description, which included her description, had been validated by the teacher. In this episode, Mr. Green took on the position of orchestrator, behavior manager, and teller while Michelle took on the position of mathematical authority.

Mr. Green: Um, is there somebody that thinks they could get it in the right spot? Put it right in? Lydia you want to give it a shot?

Lydia: It's on the scatter plot. The line that follows the general direction of the dots while, um, following [inaudible] possible and hitting as many dots as possible.

Mr. Green: James, Mike, shh. That was really good. Would you be kind enough to read it one more time?

Lydia: It's on a scatter plot, and the line is following the direction of the dots while hitting the middle and hitting as many dots as possible.

Mr. Green: Did everybody hear that? [Michelle very agitated and talking to the people around her] So, she's kind of taken several of the things that have been said, making sure that . . . K? Lydia, could you come up and like draw where it really should be up here? [Michelle listens to and watches Lydia present]

Now consider the negotiation of these positions. First, Mr. Green took on the position of orchestrator when he invited Lydia to come draw the line of best fit. Because Lydia gave a description instead, she pressed him to continue on with the discussion that he had just tried to wrap up. Mr. Green then stepped into the behavior manager position to make sure students were

listening to the conclusion of this discussion. In order to wrap up the discussion, the teacher took on the teller position by validating Lydia's response with "that was really good." Again, he was validating her for including other students' descriptions and not on the total correctness. By asking Lydia to read it again, Mr. Green took on the orchestrating position again. In his concluding statement, he took on the pseudo-orchestrating position by expressing what was good about her response. However, Michelle only paid attention to the validation on the description of "hitting as many dots as possible," which was her description of the line of best fit. Michelle was agitated that Mr. Green validated Lydia when he had pressed her to step out of the mathematical authority position for a similar response. With this indirect validation from Mr. Green, she took on the position of mathematical authority. Therefore, in an attempt to wrap up the discussion, the teacher inadvertently laid the foundation for a larger conflict later.

In terms of implications, the yielding to pressures caused problems for the teacher and student. For the teacher, Mr. Green resisted telling students what description he was looking for but he did tell a student that her description was good. In the attempt to wrap up the discussion, the discussion was prolonged, which becomes apparent in the next episode. For the student, this incident along with what had happened previously caused Michelle to take on the mathematical authority with even more fervor than before. She felt she had validation from the teacher that her description was right. As such, Michelle would not let the discussion end until it reflected her conception of the line of best fit. This resistance put even more pressure on the teacher to take on the teller position to end the discussion with a definitive conclusion in a timely fashion.

Episode 4: The unyielding, validated mathematical authority. In the following episode, the conflict between Michelle and Mr. Green manifested itself. When Lydia went to the board to draw the line of best fit, the students (Mike in particular) challenged her placement. She

adjusted the placement of the line until the majority of students agreed it was in the right place, but it no longer connected as many dots as possible in the scatter plot. After she had adjusted the line of best fit, Michelle challenged her placement based on Michelle's "validated" definition of the line of best fit, which is where this episode begins. In this episode, Mike challenged Michelle and Mr. Green expressed his concern about the amount of time the discussion was consuming. In this episode, Michelle took on the position of mathematical authority and assertive peer while the teacher took on the position of orchestrator.

Michelle:	[Raising her hand] No!
Mr. Green:	Michelle, what's wrong?
Michelle:	[Turning to face Mike] Mike, listen. There's . . . that thing is only touching two dots. The point is trying to get as many . . .
Mike:	[Inaudible]
Michelle:	That's the point!
Mike:	[Inaudible shouting]
Mr. Green:	Good. Good question here. Does it matter if it hits any dots? Mike's saying no!
Mike:	No, say it was like . . . Can I go up there?
Mr. Green:	Sure, come on up.
Michelle:	Go Mike!
Mr. Green:	We've got . . . we've got one more minute to discuss this then we need to move on.

Consider the negotiation of positions here. Michelle (as a mathematical authority) started by helping the teacher to maintain his position as orchestrator by raising her hand. As the orchestrator, Mr. Green asked Michelle to express her concern. Continuing in the position of mathematical authority, Michelle directed her comment to Mike (and not the teacher) to address his misconception of the line of best fit having to split the dots in half. She felt she had the

teacher's support on this based on the validation in the last episode. By engaging in a debate with Mike, there was pressure for Mr. Green to slip into the teller position. He tried to address the misconception in the orchestrator position by asking the question to the class. In an attempt to continue in his orchestrating position, Mr. Green accepted Mike's request to present a counterexample to Michelle's definition. By allowing Mike to present, it was apparent that he was trying to press Michelle out of the mathematical authority position. Michelle then took on the assertive peer position to taunt Mike for presenting an answer that she thought was wrong. At the end of this episode, it is clear that the teacher also felt pressed for time. He tried to press Michelle out of the mathematical authority position quickly to wrap up the discussion with a correct conclusion while maintaining the orchestrating position at the same time.

At this point, both the teacher and the student were trying to maintain their positions. With the pointed yet open question "does it matter" and with allowing Mike to challenge Michelle, Mr. Green tried to challenge Michelle's mathematical authority position. He was hoping that a class consensus as well as a powerful counterexample would be enough to press her from the mathematical authority position without having to take on the teller position. From this episode, Michelle felt she had the teacher's validation. The fact that the teacher allowed Mike to challenge Michelle may have pressed her, but she was convinced that she had an answer that was validated by the teacher. Because the teacher allowed her to challenge Mike, she could have seen the teacher allowing Mike to challenge as an opportunity to fix Mike's misconception of the line of best fit. Therefore, both the teacher and the student were in stalemate while maintaining their positions of orchestrator and mathematical authority.

Episode 5: Deterioration of positions. In this episode, the student was finally pressed out of the mathematical authority position but at the cost of the teacher taking on the teller

position and the student taking on the distracted student position. Before this episode, when Mike presented, he drew an example where there were two parallel lines of points with the line of best fit sitting between the points. Michelle tried to find something wrong with his example since she was so convinced that she was right. She claimed that it was not a scatter plot. In the interest of time, the teacher eventually told her that it was a scatter plot. While she tried to find something else wrong with it, the teacher allowed another student to challenge Michelle's challenge. After this student presented, the teacher said, "Our time's up," which showed his desire to wrap this discussion up quickly. By the time he said this, the discussion had not come back around to support Michelle's challenge. Since the teacher had allowed two students to challenge her now without showing any signs of supporting her, she requested to know which answer Mr. Green advocated, which is where this episode begins. The teacher finally yielded to telling her she was wrong in an attempt to end the discussion with a definite conclusion since the time was up. In this episode, the student took on the positions of uncertain student and distracted student while the teacher took on the positions of teller, pseudo-orchestrator, and behavior manager.

Michelle:	So the goal is not to hit as many dots as possible?
Mike:	No!
Students:	[Chattering]
Mr. Green:	No, it . . . we . . . yeah , we probably need to say something like it kinda depends, right?
Students:	[Chattering]
Mr. Green:	Sometimes [Michelle takes notes on her warm-up] . . . sometimes you might want to try to hit several of them, but is that always your goal?
Students:	[Chattering]
Mr. Green:	No.

Students: [Chattering]

Mr. Green: That's why . . . Andres . . . Mike. . . [Michelle opens her sketch book and starts drawing in it] That's why I really wanted to push on that for a minute because we need to make sure that you are clear.

Now consider the negotiation of positions. By asking the teacher to validate an answer, Michelle took on the position of uncertain student. The teacher took on the position of teller by telling her "no," but then he took on the pseudo-orchestrator position by hedging and telling her that it depended and asked the class to validate the answer. After the class responded, he was pressed into the behavior manager position to make sure that students were listening to his conclusion. He then justified the length and content of the discussion by implying that everyone was "clear" about the definition of the line of best fit. After Michelle was pushed out of the mathematical authority position, she took on the position of distracted student. She pulled out her sketchbook to doodle and her attention was no longer focused on the teacher or on the mathematics.

By the end of this episode, the deterioration of the teacher and student positions has been mapped over the course of these episodes. By deterioration, I mean the participants ended up in less desirable positions. The teacher trying to be an inquiry-based instructor slipped into a position that supported traditional instruction. The teacher started as an orchestrator, but as he tried to wrap up the discussion in a timely manner with a correct conclusion, his attempt to maintain the orchestrator position deteriorated into the teller position. The student slipped from a position where she was engaged in the mathematics to one where she was not. Michelle started as a mathematical authority, but as she tried to maintain this position based on what she thought was validation from the teacher, her attempt to maintain the mathematical authority position deteriorated into the distracted student position. Therefore, the teacher's failure at inquiry-based instruction was linked to Michelle's failure at inquiry-based mathematics.

Episode 6: Reclamation of positions. In this final episode, the student and the teacher repaired their relationship and reclaimed their positions through the negotiation of positions. Reclamation refers to the teacher and student coming back to engaging in inquiry-based instruction and to engaging in mathematical activity. Toward the end of the class period, the teacher used a task where he read off the names of famous people, the students individually recorded how old they thought they were, and then the teacher revealed their actual age so students could analyze how good they were at guessing ages by plotting the data. At first, as the teacher started reading off the names, Michelle would deviate from her doodling only long enough to write down the name of the person and her guess. As the guessing game continued, Michelle returned to her doodling less often and for shorter periods of time. When the teacher started reading off their actual ages, the students were intrigued by how close or far their guesses were to being right. Michelle was so intrigued that her worksheet completely covered her sketchbook and she did not return to doodling for the rest of class. This shows the power of tasks in that it was powerful enough to pull Michelle out of the distracted student position. After the students collected their data, the teacher asked the students to graph the data, so they could analyze how good they were at guessing ages. With this age guessing activity, Mr. Green invited Michelle to take on a position other than engaged listener. The shift in activity from doodling to engaging in and working on the task is apparent. While working on the task, she was focused on the mathematics. Because there were no speech acts and no access to her work, this position was difficult to identify, but I call this hypothesized position the mathematical reasoner for now.

Instead of asking the teacher how to graph the data, Michelle asked a very specific question that showed that she engaged in the mathematics at a deeper level, which is where this episode begins. Mr. Green showed that he valued her question by using it as a discussion

question. By doing so, Mr. Green and Michelle repaired their relationship and reclaimed better positions. In this episode, the teacher took on the orchestrator, behavior manager, and teller position while the student took on the dependent student and engaged listener position.

Michelle: Which one do we put on the x-axis?

Mr. Green: Ah, which would we want to put on the x-axis?

Students: [Chattering]

Student: Guess.

Student: Names.

Student: Person.

Mr. Green: We have a lot of people shouting and not anybody raising their hands. Shh. How do you decide which one goes on the x-axis? Jason.

Jason: Person.

Mr. Green: Person? We are not graphing person. What are the things we are graphing?

Student: Guess.

Mr. Green: Guess and?

Student: Age.

Mr. Green: Actual. Right. We've got their guess and their actual. Which one of those should go on the x-axis?

Consider the negotiation of positions in this episode. By asking about the independent variable, Michelle took on the position of dependent student. Mr. Green then took on the position of orchestrator by asking her question back to the class. By using her question as a discussion question and with the tone of his voice, the teacher showed that he valued her question. Because the students start shouting out answers, they pressed the teacher into the behavior manager position. As a behavior manager, he told students to raise their hands and to be quiet. He probably did this to ensure that the whole class could hear the conclusion of the

discussion for repetition was not something the teacher liked to engage in. By asking the students a slightly different question about how to decide what "goes on the x-axis," the teacher took on the position of orchestrator. When the student he called on gave an incorrect answer, the teacher (in the interest of time) took on the teller position to make sure the discussion focused on the variables that were actually being graphed. By asking for the variables that were being graphed, the teacher took on the orchestrator position again. Once the students provided the teacher with the variables he was looking for, he took on the position of teller to validate the students. Again, he took on the teller position to make sure the discussion ended in a timely fashion with a definite conclusion because class was almost over. When he repeated the original question about the independent variable, the teacher took on the position of orchestrator again. After Mr. Green presented Michelle's question to the class, Michelle took on the engaged listener position. She did not contribute to the discussion very much, but her attention was fixed on the teacher and students who were talking. Therefore, Mr. Green and Michelle were repairing their relationship.

Because of the nature of the task, Mr. Green and Michelle were able to reclaim their more desirable positions. Mr. Green was able to take on the orchestrator position again. Because of the eminent end of class, he took on the position of teller along with the position of orchestrator to make sure the student got to the right conclusion before the end of class. Michelle, on the other hand, stepped out of her distracted student position and into the mathematical reasoner and the engaged listener positions. The mathematical reasoner and the engaged listener foster learning better than the distracted student position. This episode suggests inquiry-based instruction can help participants reclaim positions and repair relationships.

Discussion

From these results, I can summarize the important points and work towards answering my research questions. Before talking about my results in terms of my research questions, I briefly summarize some of the characteristics of positions. Next, I focus my discussion on how these results help answer my two research questions, which focused on how students were positioned and how the positioning leads to student failure. I recognize that some of these positions may facilitate learning, but because of the nature of my research questions, I just focus on how positioning relates to student failure in inquiry-based classrooms. To evaluate the student failure, I focus on the implications of positions and the negotiation of positions on learning. Each position and the associated activities have implications for the learning that occurs in the classroom. The positions of the student are very much influenced by the teacher and vice versa. As I mapped in the previous episode, the negotiation of positions can result in participants taking on positions that can even hinder learning. During the negotiation of positions, there are certain factors that press the participants into certain positions and out of other positions. Therefore, these results are significant to the learning in an inquiry-based classroom.

Summary of characteristics of positions. In a mathematics classroom, there are many positions and the positions are intricate and complex in that positions can be dynamic or resilient to change. During a single segment of discourse, the participants can move between positions rather frequently. For example, think about how quick the teacher was to take on the behavior manager position and how quick he was to return to the position he had before that. In other situations, positions can be more resilient to change and seemed to be a default position that a participant would continue to return to after taking on other positions. Consider how difficult it was to press Michelle out of the mathematical authority position. She might have taken short

detours to positions like the assertive peer, but she would return to the mathematical authority position. Therefore, positions in and of themselves are complex in that they are changeable and yet partially stable for short periods of time.

Throughout these episodes, there were three themes that transcended the events. First, in an inquiry-based classroom, there is a lot of pressure between the teacher and students as they try to press each other into particular positions. The students kept trying to press the teacher into the teller position (by actively seeking for it or by defending a wrong answer) and the teacher would press them back to maintain his positions. Second, in this particular classroom, Michelle was very dependent on validation. She would actively seek for validation or wait for the teacher to validate answers. Once she had validation, she would resist being pressed out of the mathematical authority position, even if she was wrong. For Michelle, receiving validation was a way that she could protect her social status. With validation, she could avoid being wrong in front of her peers. Third, these issues of pressure and dependence on validation can press the teacher and student into less desirable positions (deterioration) but the participants can come back into more desirable positions (reclamation). These episodes mapped the teacher going from an orchestrator, which supports inquiry-based instruction, down to a teller, which supports traditional instruction. Also, these episodes mapped the student going from mathematical positions like uncertain student, dependent student, engaged listener, and mathematical authority down to a distracted student. However, the teacher was able to regain his positions that support inquiry-based instruction and the student was able to regain her mathematical positions. Therefore, there is much to learn by studying positioning.

Implications of individual positions. Each of these positions, whether more resilient to change or more fleeting, had an impact on learning. In this part, I focus on the student positions

and their impact on failure. I talk about the activities of the student positions and how they did not facilitate learning in an inquiry-based classroom. I do not talk about the teacher's positions' impact on learning alone because that is determined by the actual negotiation between student and teacher positions and what position the student ends up taking on, which I discuss later. Therefore, positioning is important to understanding failure in an inquiry-based classroom.

First, consider the impact of the uncertain student position where the primary activity is seeking validation from the teacher. Because of the student's dependence on the teacher, this position facilitates only limited learning. From her perception, she can only progress in her learning as she obtains the validation she is looking for. She does do some of the thinking, but because of her dependence on an external source for knowledge, the learning is limited to what the teacher tells her is correct.

Second, consider the impact of the dependent student position where the activity is seeking instruction from the teacher and smarter peers. The learning is even more limited than with the uncertain student position. In this case, the student is now completely dependent on the teacher or a smarter student and she lets them do most of the thinking about mathematics. Again, from her perception, she learns when she gets that instruction. However, she is again dependent on an external source, so the learning that can occur when a student takes on the dependent student position in an inquiry-based classroom is limited.

Third, consider the impact of the assertive peer position where the activity is non-mathematical and she engages in making social judgments. While in this position, learning mathematics is not her focus. However, this position did influence the more mathematical positions that she did take on. How and why she engaged as an uncertain student, dependent student, mathematical authority, distracted student, and engaged listener was determined by the

social tasks she was trying to accomplish, which centered around protecting her social status. Therefore, the activities of the assertive peer may not have had a direct impact on learning mathematics, but it's relation to other positions in terms of protecting social status could have a negative impact on learning and cause failure.

Fourth, consider the impact of the mathematical authority position where the activity was sharing and defending her answers. While in this position, she was more invested in her answers and her utterances indicated that she has thought about her answers enough to defend them. In this way, this position could actually facilitate learning. However, as seen with that class period, she could embrace an incorrect conception as mathematical truth because of perceived validation. When this was the case, her choice of topic to debate and her dependence on validation could further lead to failure. For her, definitions and adhering to definitions was important because when she argued, she was arguing how well something fit the definition or not. In this particular class period, she took on the mathematical authority a few times to debate mathematical conventions and definitions. As such, logic was not enough to dissuade because it was a convention, and she got discouraged with the mathematics when she found she was wrong. From these results, the role of validation in this classroom had a huge impact on learning. Once Michelle received validation from the teacher or from multiple peers, then she would make the idea her own, defend the idea fiercely, and not yield to other possibilities until the teacher told her she was wrong. Again, she used validation to protect her social status. When she was told she was wrong, she would step out of this position and out of the mathematics. Therefore, this position could facilitate learning but the choice to argue conventions and the stubborn reliance on received or perceived validation could be detrimental to learning and lead to failure.

Fifth, consider the impact of the position of distracted student where the activity was doodling or socializing. Because the student disengaged from the mathematics, learning was very minimal. By allowing herself to be distracted, she might miss some big mathematical ideas, which would cause her to do poorly on assignments and tests. Therefore, the distracted student position could be detrimental to learning and lead to failure.

Sixth, consider the impact of the engaged listener position where the activity was listening attentively to the mathematics discussion. For this position, the learning depended on the type of listening that the student was engaged in. If Michelle listened attentively in order to find the right answer, then the learning was probably minimal. Therefore, the engaged listener position could lead the student to fail in this inquiry-based classroom.

Now, in contrast, consider the impact of the hypothesized mathematical reasoner position where the activity was silently working on mathematical tasks. The mathematical reasoner position was so private that I could not get enough data to fully identify it, so it is a hypothesized position. When the student was vocal in her other positions, she sought for validation in order to protect her social status. The speech acts of these positions that I studied were public, and Michelle could not take on any of these positions without affecting her social status. In contrast, with the hypothesized position, Michelle could engage silently without any threat to her social status. This hypothesized position has potential to facilitate learning because Michelle could engage in the mathematics and be right or wrong without worrying about what her peers thought of her. Of all the positions, this position is the closest to supporting inquiry-based instruction. This suggests that Michelle could only engage in positions that do not threaten her social status.

Implications of negotiating positions. However, the student's positions did not work alone. The teacher's positions influenced student's positions and thus student learning in two

ways: first, through the recognition of certain student positions, and second, through the activities of the teacher's position. The student could take on any position in relation to the teacher's position but some positions were more likely (and some less likely) and reinforced by the teacher's positions. When the teacher took on a position, he could recognize lots of different student positions. However, there are two different types of recognition: stable and unstable. With stable recognition, the teacher (in a particular position) would support the student taking on a particular position. In this way, the teacher and the student continued in these positions together. With unstable recognition, the teacher (in a particular position) would push the student to take on a new position. Furthermore, the activity of the teacher's position may have also had an impact on student learning. As a result, the teacher's positions influenced her learning and thus her failure, either through recognizing positions, activities of his positions, or both. Because of my research question, I only focus on the teacher positions that could directly contribute to student failure in the inquiry-based classroom.

First, the orchestrator position recognized some of the student positions, which can impact learning along with the activity of the position. As stated before, the teacher as an orchestrator could have recognized lots of student positions, but the recognition is stable in some cases and not stable in others. While the teacher took on the orchestrator position, he sought for students who would engage in defending, challenging, and questioning answers. Hence, as an orchestrator, there may be stable recognition of the mathematical authority position. In terms of the impact on learning (from the last section), the mathematical authority could be detrimental to learning. On the other hand, when the student took the uncertain student position while the teacher was in the orchestrator position, he would try to press the student into another position. Thus, with the uncertain student position, there was not stable recognition by the orchestrator. In

terms of student learning, the orchestrator trying to press her out of that position could be beneficial to learning if the student takes on a position that facilitates learning. If the student maintains the uncertain student position or takes on positions like dependent student, then this could be detrimental to learning. As for the activities of the orchestrator, the teacher was really trying to get students to use logic to judge the correctness of the mathematics, which can be beneficial to learning if students learn to be more independent from the teacher and the textbook. Therefore, this position may have a negative impact on learning, depending on how the student responds.

Second, the pseudo-orchestrator had a mixed impact on learning. While in this position, the teacher sought for students to provide answers to his leading questions and for students to listen to the answers. In this case, there could have been stable recognition of the student as an engaged listener who listens for the right answer. Because of the student's dependence on the teacher to ask the questions to point to the right answers, the engaged listener position (that listens for the right answer) could hinder learning. On the other hand, there could have been unstable recognition of the mathematical authority position because the teacher did not want students challenging answers at this point. He would try to press students out of this position. As for the activities of the pseudo-orchestrator and their impact on learning, the pseudo-orchestrator did not facilitate as much learning in an inquiry-based classroom as the orchestrator position did. The use of leading questions and challenges of wrong or incomplete answers could have made the student more dependent on the teacher (instead of logic) to tell them what was right or wrong. Therefore, the pseudo-orchestrator can be detrimental to learning in an inquiry-based classroom.

Third, the administrator position was indirectly related to learning. While engaged in the administrator position, there was stable recognition of other positions (like students leaving early or the office calling him) that were seeking non-mathematical information about the class from him. Since this does not fit with any of the student positions, the teacher as an administrator did not have stable recognition for these student positions. Since the teacher as an administrator did not try to push the student out of certain position, there was no recognition that was unstable. In terms of the activities of administrator in the middle of class, the administrator had the power to interrupt discussions and prevent certain misconceptions from being corrected and certain conclusions from being made. Therefore, this position has the potential of being detrimental to learning.

Fourth, the teller position was detrimental to learning inquiry-based mathematics. While in the teller position, the teacher sought for students to receive his validation and direct instruction. As such, there was stable recognition of the uncertain and dependent student position. This position facilitates limited learning. On the other hand, there was unstable recognition of the mathematical authority position because the teller did not want students defending their answers. Since it depended on what position Michelle decided to take on in response to this pressure, this may or may not have a negative impact on learning. In terms of the implications of the activity, the activities of the teller (giving validation and direct instruction) facilitate only limited learning since the students are dependent on the teacher. Therefore, the teller position hindered the learning in an inquiry-based classroom.

From this single class period, it was evident that the student took on positions that contributed to her failure in the classroom (like the distracted student) and that the teacher took on positions (like teller) that contributed to his struggles and the student's struggles with inquiry-

based instruction. Although some of Michelle's positions could facilitate learning, she did not take on many positions that fully supported learning in inquiry-based mathematics. When the teacher slipped from the orchestrator position he felt like an ineffective instructor, but he believed so much in inquiry-based instruction that he would not leave it completely. However, this class period also illustrated the power of inquiry-based instruction as the teacher was able to pull Michelle back to positions that facilitated more learning in the classroom.

Some factors in the classroom (like student behavior, student contribution, social status, etc.) have more power to press the teacher and the student out of positions than other factors. So, what are these factors that caused the teacher and the student to step out of their positions and slip into positions that caused them both to fail? This question is addressed in the next chapter.

Chapter 5: Position Pressures

As I analyzed the negotiation of positions, I noticed that certain factors were strong enough to press the teacher and the student to step out of certain positions and into other positions. I went through and identified places where the teacher or the student were being pressed to change positions and then I identified what factors were present that may have pressed them to change positions. Some of these factors were stronger than others and a change in position did not always occur. I called these factors that pressed the participants to change their positions Position Pressures or PoPs for short. I also noted that when some of these PoPs were present or when enough of them were present the participants stepped into positions that contributed to a sense of failure (failure as an inquiry-based teacher or failure as a mathematics student) or positions that were beneficial to teaching and learning inquiry-based mathematics. Just as the student can fail to engage in the learning activities of inquiry-based mathematics, the teacher can fail to engage in the teaching activities of inquiry-based mathematics. Therefore, when the teacher fails to maintain inquiry-based instruction, he fails to be an inquiry-based instructor.

In this chapter, I identify the PoPs of the teacher and the student. Some PoPs caused the teacher to change positions. There were PoPs that caused the teacher to slip into positions like the teller while other PoPs helped the teacher take on the position of orchestrator. I further talk about the implications of teacher slippages for the teacher and the student. Other PoPs caused the student to change positions. There were PoPs that caused the student to slip into positions like the distracted student while other PoPs helped the student take on positions like engaged listener or mathematical authority. I also talk about the implications of student slippages for the student and the teacher. From this analysis, I found that the PoPs have an impact on learning.

Teacher PoPs

Timely coherence. During an interview, Mr. Green told me that he liked students contributing to discussions because then misconceptions come out and the class moves as a group towards what is acceptable, which was worth "a few days of chaos." This shows that students' mathematical understanding was important to Mr. Green, and he was willing to take the time to make sure that students had correct understanding of mathematical concepts and procedures. From observations, Mr. Green tried to foster this understanding by making sure that discussions ended with definite conclusions that the class could agree on. Mr. Green did not worry about the amount of material being covered as much as coming to a clear conclusion about a misconception or mathematical concept. During class, the teacher's frequent references to time and the lack of time showed that it was important to him for these discussions to end with a definite conclusion in a timely fashion, which is what I refer to as timely coherence.

Because Mr. Green valued timely coherence so much, factors that prevented this could push him from one position to another. One such factor, or PoP, was unproductive student comments. During an interview, the teacher talked about the stress of inquiry-based instruction. He said he sometimes tried to strategize on who to call on, but there were times when Mr. Green called on a student and his or her contribution was something that he did not anticipate. Hence, these comments did not take the class discussion in the direction he wanted them to go and thus were unproductive to reaching that definite conclusion of the discussion. These unproductive comments frequently caused the teacher to slip from an orchestrator to a pseudo-orchestrator or teller, or from a pseudo-orchestrator to a teller. For example, in Episode 6 from Chapter 4, when the end of class was quickly approaching and Michelle asked the important question about which variable to use for the x-axis, one student suggested person. This was an unproductive comment

because person was not even a variable for what the teacher had asked them to graph. In the interest of time, the teacher slipped out of the orchestrating positions and took on the position of teller and told the student "no" so that students could discuss and decide between the two variables it could be. Therefore, unproductive student comments are a PoP that can press the teacher out of certain positions and into others.

Another factor, or PoP, that could influence the timely coherence of a discussion was discipline. During an interview, the teacher talked about how he wanted students to contribute but "one student off base," or being disruptive, could be problematic. This explained why he disciplined students who were talking when he called on someone to present. The teacher wanted to make sure that students took the discussion time to talk about mathematics and to listen to the definitive conclusions that the class came to. These discipline issues caused the teacher to slip from any other position into a behavior manager position. An example of Mr. Green slipping into a behavior manager position is in Episode 1 from Chapter 4, when the teacher called on Steven to contribute and Mike started answering instead. Mr. Green slipped from the orchestrating position into the behavior manager position so that this student could be heard and so that other students would not think it was acceptable to start talking as well. After Mike complied, the teacher stepped back into the orchestrator position. Therefore, discipline issues are a PoP that can press the teacher to change positions.

In terms of implications, the unproductive student comment PoP and the discipline PoP can be enough to press the teacher in choosing the teller position. In interviews, he expressed that implementing inquiry-based instruction was hard. This slippage could be why the teacher felt this way. When the teacher slipped into the teller position, there were various implications for students. Some students may have waited for this slippage to happen to get their answers

from the teacher. Other students may have actively pressed the teacher to take on the teller position when he was not in that position since they knew that he could slip into that position. Also, there could have been some students who supported the teacher as an orchestrator when he had taken on the teller position, and as such there could have been problems when the teacher engaged in the activity of teller when students were expecting a different activity. This contradiction in activity could further teach students to support the teacher as a teller. From the literature, there is evidence that the teller position was detrimental to learning, which Mr. Green had learned from experience and was why he tried to avoid such activities.

Social status. Another difficulty that the teacher mentioned during an interview was that some students were more concerned about the social aspects of the classroom than the mathematics. He also talked about being more lax on discipline to encourage more students to participate and contribute, but some students interpreted this as an opportunity to socialize. Classroom management and the negotiation of classroom norms was something the teacher wanted to improve on so that students would better know what acceptable behavior in his class was. From this interview and from classroom observations, it was apparent that the pursuit for social status was significant and had an impact on the teacher's teaching and the students' learning. As Michelle demonstrated with the assertive peer, the students (some more than others) negotiated social status with other students (like when she told Mike to speak up in the description of the mathematical authority position in Chapter 4) and even with the teacher (as when she challenged his claim that her hand was not raised in the description of the assertive peer position in Chapter 4). However, Mr. Green did not allow students to negotiate the activity of mathematics, which was evidenced by the fact that students were not able to persuade the

teacher to get off topic. This shows that the teacher had some success with classroom management since he did not allow students to press him in this way.

Because students' pursuit for social status could hinder the learning of mathematics, discipline issues for the teacher again were a PoP. When students were having their own conversation about mathematics, he generally let this slide. However, when students were trying to assert or improve their social status by being entertaining, for example, the teacher would commonly slip from whatever position he was in and take on the behavior manager position. During the age guessing activity, Mr. Green had to frequently stop while reading the names of famous people because of the excessive amount of student commentary, which the segment below exemplifies. Mike in particular was trying to be entertaining, and this was his second or third attempt, so Mr. Green felt pressed to step into the position of behavior manager to correct Mike's behavior.

Mr. Green: The next one is Jennifer Aniston.

Mike: Oh, she's even hot!

Andres: I don't even know who that is.

Mr. Green: Shhh. Mike. We don't need your opinion on each one.

So when the pursuit for social status interfered with the mathematics being taught, Mr. Green would slip into the behavior manager position.

Even when students were contributing to discussions, they would compete for social status using the mathematics, which was a PoP for the teacher. This competition was a key characteristic that I noticed during my observations. Towards the beginning of the semester, Mia and Jim were the primary contributors of discussion and would compete for the right answer. Towards the end of the semester, Mia and Jim were no longer in class and so Mike and Michelle took their place in competing for social status by competing for the right answer. Recall the

examples of mathematical authority, like in Episode 4. Michelle would choose an answer and Mike would challenge her. They would continue debating until either enough students or the teacher validated one of their answers. While debating, Michelle would commonly step out of the mathematical authority position and into the assertive peer position, like when she told Mike "that's the point!" This suggests that the mathematical authority was greatly influenced by social factors and thus these students were seeking (at least in part) for social status by debating the mathematics. Logic was not enough to convince either one of the correctness or incorrectness of their answer. They relied on an outside source to choose the winner, which is where the PoP on the teacher comes in. Since the students were not coming to a conclusion or the right conclusion, the teacher would step out of his orchestrator position and into a pseudo-orchestrator or teller position. Therefore, the students' competition for social status was enough to press the teacher to choose to step out of the orchestrator position.

In terms of implications, these social status PoPs have the same implications as the timely coherence PoPs. The teacher slipping from the orchestrator position to the teller position made the teacher feel ineffective and the teller position was detrimental to student learning. Therefore, these PoPs can be detrimental to both the teacher and the student.

Structure of school system. During interviews, Mr. Green expressed his stress related to the structure of the school system. One stress was the lack of support he received from fellow teachers who believed that his style of teaching was "fluff." Another stress was the pressure he felt from the school to assess traditionally. As a teacher, Mr. Green not only had to cater to the learning needs of the students, but also to the policies and procedures of the school system. As such, Mr. Green was not only juggling students but also school procedures. He could take care

of some of these procedures outside of class time, but others required attention during class. This was the reason that he took on the administrative position.

Because the teacher had to take care of administrative issues during class, administrative interruptions (which were a part of the school structure) were PoPs for the teacher. The teacher was expected to take time to prepare students who were leaving early, to accept notes from office assistants, to pause class for announcements over the intercom, and to answer phone calls from the office. Whenever these issues arose, the teacher felt pressed to step out of whatever position he was in to take on the administrator position. These interruptions can be detrimental to discussions in inquiry-based classrooms. As mentioned previously in Episode 2 from Chapter 4, Michelle's misconception of the line of best fit was dropped when the discussion was interrupted by a phone call. Addressing misconceptions and developing mathematical understanding across the class was why Mr. Green valued discussions. However, interruptions such as these prevented misconceptions from being fully addressed and understanding from being fully developed. During another class period, there was another phone call during a critical part of the discussion. This time, instead of answering the phone call, Mr. Green focused on finishing the discussion. When Mr. Green did not answer the phone, an office attendant came down to his classroom to find out why he had not answered the phone. This shows the pressure on the teacher from the school structure to take on the position of administrator during class. In traditional instruction, the teacher has a plan as to what content is to be delivered and student thinking is not as important, so interruptions in a traditional classroom is less detrimental because the teacher can pick up where he or she left off in the content. In contrast, interruptions of inquiry-based discussions can be more detrimental to student learning.

In terms of implications, the interruptions were problematic for both the teacher and student. For the teacher, when a misconception was dropped or a concept was not fully developed, the teacher may have more work to do, such as in the class period of the previous chapter. Since Michelle's misconception was not fully addressed, the teacher had to deal with that on a larger scale later. At other times, he may have to revisit a topic with another discussion later because it was not addressed sufficiently in a previous discussion. This could add to the sense of failure for the teacher since the students did not seem to grasp these concepts after a lengthy discussion. For Michelle, these interruptions could add to her sense of failure at mathematics. When she did not "get it," it could add to her sense of dislike for mathematics. She told me in interviews that she liked mathematics when she felt like she "got it." As the previous chapter showed, the failure to address the misconception when it came up caused an even bigger conflict later when Michelle was convinced that she had the right answer. As a result, she took on the position of distracted student. Therefore, the PoP of interruptions impacts the teacher and student's sense of failure in inquiry-based classrooms.

Positive PoPs. While I was studying these PoPs in terms of the deterioration of positions, I found that there were also positive PoPs that helped the teacher take on positions that made him feel like an effective inquiry-based instructor, such as orchestrator. When students would raise their hands and speak when called on, they allowed the teacher to take on the orchestrator position. There were times when students were speaking out of turn, but one student would raise his or her hand so the teacher called on him or her because then he was able to maintain his orchestrating position.

At other times, students supported Mr. Green as an orchestrator when they would say what the teacher wished he could say. For example, in the description of mathematical authority

in Chapter 4, Mike challenged Michelle's answer for the slope by saying "Wait, 2 over 3, what do you mean?" Even though Mike did not raise his hand, Mike's utterance was the challenge to Michelle's incorrect answer that Mr. Green felt he could not give as he tried to maintain his orchestrator position. This is evident from the teacher drawing attention to Mike's challenge. So, when students engaged in teacher activities such as challenging thinking, they were supporting Mr. Green as an orchestrator.

However, Mr. Green did not always take the opportunity to take on the orchestrator position when a student supported it. In this segment, Mike provided another description of the line of best fit, which was problematic, but the teacher did not challenge his description. Michelle did speak up to challenge Mike's description and thus supported Mr. Green taking on the orchestrator position, but Mr. Green did not take on the orchestrator position, as this segment shows.

Mike:	[Shouting out] Starting at the left and angling down to the right.
Mr. Green:	That's part, that's part of it.
Michelle:	[Without raising her hand] But maybe the angle's going this way. [motioning a line with a positive correlation with her hand, facing Mike]
Mr. Green:	Right, but if we had a . . . if we had a positive correlation then you've got a problem with saying that. Right?

By saying "that's part of it," Mr. Green took on the pseudo-orchestrator position. However, Michelle recognized the problem with the description. When Michelle took on the position of mathematical authority and emulated the teacher by challenging Mike's incorrect statement about the line of best fit, she provided an opportunity for the teacher to take on the orchestrator position. However, because of the presence of the issue of timely coherence, the teacher did not take on the position of orchestrator again. This shows that the PoPs associated with timely

coherence, social status, and the structure of the school system can be more powerful than the positive PoPs when present.

Student PoPs

Social status. During interviews and classroom observations, I found that social status was a huge concern and PoP for Michelle. Even when I asked the teacher about why she had such a low grade in the class, he told me that he thought she was more concerned with the social aspects of the classroom. However, I found this issue of social status to be a very complicated issue in the classroom. She seemed to be trying to preserve a positive self image in front of her peers. When mathematics was not the focus, she would do this by taking on the position of assertive peer. When mathematics was involved, she did this by taking on the dependent student position, uncertain student position, or the mathematical authority position. By taking on the position of dependent student while presenting, she admitted that she was confused by the mathematics and needed help. This way she was neither right nor wrong. By taking on the position of uncertain student, she showed that she understood enough mathematics to determine an answer but she expressed uncertainty to protect herself in the case she was wrong, as in Episode 5 from Chapter 4. By asking a question, she showed what she understood and what she was uncertain about. If she was right, she could then take on the position of mathematical authority. By taking on the mathematical authority position, she could show her peers that she understood the mathematics and that she was right, as the description of mathematical authority and in Episode 4 from Chapter 4. In an interview, she told me that she loved to present because then she could show the teacher that she understood. She also told me that she did not like to contribute to the discussions when she felt like most people understood the mathematics. This demonstrates that she took on the position of mathematical authority when she felt like there

were people who did not understand the mathematics. This idea of showing competence (that others appeared not to have) was a part of preserving a positive self image in front of her peers.

In terms of implications, the pressure that the student felt to maintain her social status with her peers affected her learning and Mr. Green's teaching. This pressure could inhibit Michelle's learning because she only took on certain positions at certain times to protect her social status (like taking on the mathematical authority position only when others seemed to not understand the mathematics). The resilience of the mathematical authority even inhibited learning since logic was not enough to dissuade her. There may be public positions more conducive to learning for her but they would compromise her attempts to maintain her social status. As for the teacher, the consequences associated with these student PoPs are felt in the teacher's PoPs associated with social status. Therefore, the PoP of social status is problematic for both the student and the teacher.

Teacher's movement between orchestrator and teller. Another form of PoPs that pressed the student to take on different positions was the teacher's movement between orchestrator and teller. Sometimes Michelle tried to support the teacher as an orchestrator when he had taken on the position of teller, such as when she took on the position of mathematical authority to challenge Mike's description of the line of best fit. Other times she tried to press the teacher into the teller position when he had taken on the orchestrator position. In the description of the uncertain student position in Chapter 4, Michelle pressed him twice to step into the teller position by asking him "right?" However, the teacher pressed back and maintained his orchestrator position by asking her to just present the first time and by asking the class to validate her the second time. He tried to get Michelle to take on a position where she was not seeking validation from him, but she may not have felt capable of taking on such a position since

she was uncertain. When I talked to Michelle about the teacher's behavior during an interview, she told me that he was "moody" in the sense that his actions were not always consistent. The teacher switching between orchestrator and teller could have contributed to this idea. Therefore, the teacher's movement between orchestrator and teller was problematic for Michelle.

In terms of implications, this movement between the orchestrator and teller position affected the learning and teaching in this classroom. This movement could be why Michelle adhered to the uncertain and dependent student positions (which foster limited learning) so much. Also the teacher's shift from orchestrator to teller could have caused the student to take on the position of distracted student, as she did in the previous chapter. As for the teacher, this shows why students may have made teaching inquiry-based mathematics so difficult. Therefore, this movement between positions is significant in the teaching and learning of inquiry-based mathematics.

Contested positions. Another interesting PoP that I found for the student was the idea of contested positions. With contested positions, the teacher tried to press the student into a position that she did not feel like she was capable of taking on. The teacher saw the student as capable though. It could be that the activity seemed out of reach or the social implications made it unattainable for the student. Whatever the case, the position was contested because the teacher saw the student as capable when she did not. From interviews, Michelle saw herself as an average student and she saw mathematics as a difficult subject. As such, she may not have seen herself as capable of taking on a smart student position. The following segment was a good example of this. In this segment, students were still in disagreement over Michelle's presentation of the slope of her line of best fit, so Mr. Green (in an attempt to maintain his orchestrator position) asked Michelle to present the correct answer. She expressed uncertainty

over what to write since her answer on the board was a rise of two. As the teacher continued to press her to write the correct answer, Michelle slipped into the dependent student position.

Mr. Green:	So, grab that . . . grab that pen right there, and write on top of it what . . . what it . . . what is it? Is it 2?
Michelle:	Do I put 200?
Mr. Green:	I want you to put whatever it really is. How much does it really rise?
Michelle:	[Inaudible] Yeah, if it's 2 and each square is 100 . . . [Looking at teacher and waiting for his response]
Mr. Green:	Look at it. Is it 2 or 200?
Mike:	200!
Student:	2
Michelle:	I don't know. I am getting confused. [Looks back at board then back at teacher]

To Michelle, the right answer was on the board so the teacher's request for her to present the right answer made her question her answer and pressed her to seek for validation by saying "do I put 200?" From the pressure to present from the teacher and the mixed answers from students, Michelle slipped into the dependent student position and expressed confusion. In this example, the teacher tried to push Michelle into a position where she would defend the "right" answer for the rise of the slope, but Michelle did not feel capable of taking on such a position because she did not know what the right answer was. As a result, Michelle slipped from an uncertain student position to a dependent student seeking for instruction.

In terms of implications, contested positions were problematic for both the student and the teacher. When the teacher pressed her to do something she felt incapable of, she could slip into positions like dependent student, which was not productive to learning. For the teacher, this could add pressure as he tries to figure out how to avoid the teller position and still make sure the discussion has timely cohesion. Therefore, contested positions are significant to the positions that the student and teacher takes on.

Positive PoPs. While I was studying PoPs in terms of the deterioration of positions, I found that there were also positive PoPs that helped the student feel like she "gets it." One such PoP was the value the teacher placed on her contribution, like in Episode 6. By putting value on a comment, question, challenge, or parts of a solution, the teacher often positioned her differently or could have invited her to take on a more desirable position. By so doing, the teacher gave Michelle an opportunity to preserve her positive self image in front of her peers. As a result, she might step into positions like engaged listener or mathematical authority. For example, when she asked which variable to put on the x-axis, the teacher showed that he valued her contribution and may have positioned her or invited her to take on the position of valuable contributor. As a result she took on the position of engaged listener, which is more conducive to learning than distracted student.

Another positive PoP for the student was the cognitive engagement of inquiry-based activities. The age guessing game intrigued Michelle to the point that she stepped out of her distracted student position, as shown in Episode 6. During another class period, the class was given the task to build and test the strength of paper bridges. Again, this activity intrigued her and she was actively engaged in the discussion that ensued afterwards.

A final positive PoP was peer validation. Her peers were positioning her as a correct contributor. When her peers validated her work, she moved from an uncertain student position to a mathematical authority. Therefore, there are many parts of inquiry-based instruction that encourage a student to take on positive and active positions that are more conducive to learning.

In terms of implications, there were many PoPs that were detrimental to the student and the teacher, but there were also PoPs that were beneficial to the student and the teacher. For the student, her confidence in her mathematical ability grew when such PoPs were present. This was

evident from the change in her participation over the course of the semester. For the teacher, his confidence in inquiry-based instruction grew when he saw students engaged in the activities of inquiry-based mathematics. These factors could help him maintain his position as an orchestrator. Therefore, these PoPs have positive implications for both the student and the teacher.

Conclusion

In conclusion, PoPs have a huge impact on the negotiation and slippage of positions in the classroom. Some of this slippage led to failure for both the teacher and the student. However, students learned to support the teacher as an orchestrator and the cognitive engagement of inquiry-based tasks could lead to confidence for both the teacher and the student. Therefore, PoPs have important implications in the study of positions and the impact of positions on learning and teaching.

Chapter 6: Conclusion

From this study, there are important implications and limitations associated with failure and positioning in an inquiry-based classroom. First, this study adds to the theory of positioning and student failure in an inquiry-based classroom. Second, this study has important implications for the learning and teaching of mathematics as well as for teacher education. Third, this study has limitations which can direct future research about positioning and failure in inquiry-based classrooms. In this chapter, I hope to highlight the significance of my findings and to motivate further research in this area.

Implications for Theory

As outlined in the first two chapters, there were gaps in the current research and theory that I tried to fill through my study. One gap was the lack of research about students failing in inquiry-based classrooms. Another gap was the limited lens of positioning by itself. A final gap was looking at student failure in an inquiry-based classroom through the lens of positioning built up from Gee's (2005) theory on discourse. These were the three primary gaps I tried to fill.

In my examination of the literature on student failure in inquiry-based classrooms, it was very limited, which motivated my study. With this study, I was able to find an inquiry-based classroom and a failing student to study. By so doing, I was able to document student failure in an inquiry-based classroom and look at why the student might possibly be failing through the lens of positioning. I found that the student might be failing because of the deterioration of positions or because of the PoPs that are present. In particular, I found that the factors that cause the student to fail may also be the same factors that contribute to the teacher failing as an inquiry-based instructor.

When I first looked at positioning theory, it was rather limited in the way it talked about positions. Harre and Langenhove (1999) focused on positioning occurring through language in use. However, positioning also relies on the accompanying actions of the language in use. Furthermore, in order for a participant to recognize the position, he or she needs to have access to the history of the language in use. By building up positioning theory from Gee's (2005) discourse theory, I could more fully capture the complexities of human interactions. With my results, I could show how flexible and dynamic positions are, which was not addressed with either of these theories. Additionally, I cataloged PoPs, position pressures, which directly affect the positions that participants choose to take on. PoPs are a reflection of what participants in the discourse value and what external factors are present that press the participants into taking on particular positions. Therefore, my study contributes to positioning theory.

Through this study, I tried to demonstrate how success in mathematics is more than just what a student knows. Social interactions with the teacher and thus positioning played a part in what Michelle learned, and did not learn. When it came to the line of best fit, she formulated a definition of what that was and was convinced that she was right when she perceived validation from the teacher. With this, she took on a particular position. This shows that what she learned was directly impacted by her positioning in response to the teacher's actions. Furthermore, positioning can support or undermine instruction. In this case, there was one position (a hypothesized private position) that might have supported inquiry-based instruction. However, there were a few positions that undermined inquiry-based instruction because they continually pressed the teacher to take on positions that supported traditional instruction. Based on the deterioration of positions and the PoPs, this study shows that inquiry-based instruction is difficult to implement. It is through positioning that learning is impacted and that inquiry-based

instruction is either maintained or undermined. It is plausible that student and teacher failure (or even success) is linked in an inquiry-based classroom because of positioning. Therefore, positioning is important to the learning and teaching of inquiry-based mathematics.

Implications for the Practitioner: Pre-Service and In-Service Teachers

In addition to the contributions my study makes to theory, my study also has important implications for the practitioner. There are important implications for the learning and the teaching of inquiry-based mathematics. Additionally, there are important implications for teacher education for pre-service and in-service teachers. Finally, there are important implications for the structure of the school system. So, my study contributes to both the theory and practice of mathematics education.

Because students are so concerned with protecting their social status, students need to have access to public positions (positions that are characterized by a myriad of speech acts) that facilitate the learning of inquiry-based mathematics while protecting their social status. In this particular study, students had learned to engage in activities that had the potential to facilitate the learning of inquiry-based mathematics like questioning assumptions, challenging answers, and explaining reasoning. However, the activity of seeking and using validation to protect their social status was problematic to learning. This activity protected their social status in traditional mathematics, so we need to help students take on public positions that protect their social status as well as facilitate learning in an inquiry-based mathematics classroom. By doing so, we could mitigate the effects of the social status PoP for students and provide positions that students feel they are capable of taking on, which could mitigate the effects of the contested position PoP. If students are allowed to take on positions that protect their social status, then there could be less pressure on the teacher since the affects of the social status PoP for the teacher would also be

mitigated. Therefore, facilitating such public positions would be beneficial for the students' learning and the teacher's teaching.

Currently, teacher education is limited in training teachers how to maintain inquiry-based instruction. The particular teacher that I studied was heavily involved in the field of mathematics education by participating in committees, conferences, workshops, and professional development. He also recently received a master's degree in mathematics education and made his teaching a focus of study. Hence, this teacher was informed and educated about inquiry-based instruction and still he struggled to implement it in his classroom. This research suggests teachers are not given the necessary tools needed to maintain inquiry-based instruction.

As for the implications for teacher education, my study could be a starting place for teachers to gain access to the tools needed to maintain inquiry-based instruction. Teachers (pre-service and in-service) can improve their teaching of inquiry-based mathematics by learning how to maintain their positions that support inquiry-based instruction when PoPs are present. The PoPs discussed in Chapter 5 are specific challenges that teachers face when trying to implement inquiry-based instruction. Teacher education could focus on training teachers how to maintain positions such as orchestrator when specific PoPs are present. So, teachers could learn what activities to engage in when faced with pressures such as timely coherence, social status, and the structure of the school system. By doing so, teachers may be able to avoid the deterioration to positions that support traditional instruction, like the teller position. This could help mitigate the effects of the movement between orchestrator and teller PoP for the student. Furthermore, teachers could be trained on how to incorporate positive PoPs into their teaching practices to further maintain and encourage positions (teacher and student) that support inquiry-based

instruction. Therefore, by educating and training teachers in dealing with and incorporating PoPs, the teacher and the student could be pressed less by negative factors in the classroom.

In terms of the school structure, my study shows the need for schools to more fully support inquiry-based instructors. When the teacher is pressed to take care of administrative issues during class (especially during discussions), there is potential harm to student learning as important mathematical concepts are dropped or student misconception are left unaddressed. Therefore, school structures can more fully support inquiry-based teachers by mitigating the interruptions that can occur during class time.

Limitations and Future Research

Even though this study has limitations, these limitations serve as important directions for future research. The limitations come from the very specific focus of my research problem. My research focused on one student's positioning (based on speech acts) and failure in an inquiry-based classroom. So, there are limitations on what I can say about student positions associated with few or no speech acts, about teacher failure, about solutions to the identified problems, and about multiple students, teachers, and classrooms in general.

First, I focused my research on student failure. Even though my data allowed me to draw some conclusions about teacher failure as an inquiry-based instruction because of his impact on student learning, my data collection methods were focused on acquiring data about student failure. Therefore, more research could be done to investigate why teachers fail or struggle with inquiry-based instruction and what PoPs exist for teachers of inquiry.

Second, my analysis of my data focused on the positions associated with a lot of speech acts. Because I only had a few weeks with Michelle actually sitting in front of the camera, I did not analyze what positions she could be taking on while silently participating in the mathematics,

discussions, and/or tasks. There could be more research done on these private positions where the student does not have to worry about protecting his or her social status. These positions could have implications for learning that I was not able to capture with my analysis on positions associated with a lot of speech acts.

Third, my study focused on what could be contributing to student failure. Hence, my study identified specific problems associated with positions in the mathematics classroom. The problems I identified had to do with the specific activities of positions and with negative PoPs that could be contributing to student failure in the class. More research is needed on how to address these specific problems for the student and the teacher. So research needs to be done on how to position or offer better positions for students to take on, like positions that protect social status and fit with the norms of inquiry-based instruction. Other research could focus on how to help teachers cope with negative PoPs as they try to maintain positions that support inquiry-based mathematics or on how to incorporate positive PoPs into their teaching practices.

Forth, my research focused on one classroom, one teacher, and one student. My study does offer a case example of failing in an inquiry-based classroom. However, my study and its results could be more generalizable if done for multiple classrooms with their teachers and with multiple students from different classrooms. Research that would focus on multiple classrooms and multiple teachers and multiple failing students could find common themes about positioning, student and teacher failure, and PoPs. With these common themes, we could further identify the problems that students and teachers have with learning and teaching inquiry-based mathematics and find ways to address these problems in the classroom. With this knowledge, we could also improve teacher education of inquiry-based mathematics.

Overall, my study has important implications for mathematics education even though it is limited. Positioning in an inquiry-based mathematics classroom has not been researched much, but studying positioning provides important insights about the learning and teaching of mathematics. Furthermore, we can learn much from positioning about how to improve mathematics instruction and education. Because of the importance of positioning, this research is significant and more research is needed.

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